Homogeneity tests for categorical variables

Consider two categorical variables with the same number of categories *k*. The bivariate probability distribution of these variables is represented by the matrix of probabilities

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1k} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{k1} & \pi_{k1} & \cdots & \pi_{kk} \end{bmatrix},$$

where π_{ij} is the probability that the first variable falls in category *i* and the second variable falls into category *j*.

The homogeneity test is a test of the hypothesis that the two marginal distributions are the same:

$$\sum_{j=1}^{k} \pi_{ij} = \sum_{j=1}^{k} \pi_{ji}.$$

Let

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix}$$

be the corresponding sample proportions and let $d_i = \sum_{j=1}^{k} (p_{ij} - p_{ji}), \quad i = 1, 2, ..., k-1.$ Then the Wald statistic for testing this hypothesis is $\mathbf{d}'\mathbf{A}^{-1}\mathbf{d}$, where $\mathbf{d}$ is a vector of order $k-1$ with elements $d_1, d_2, ..., d_{k-1}$ and $\mathbf{A}$ is the covariance matrix of $\mathbf{d}$. $\mathbf{A}$ is readily determined from the fact that (Agresti, 1990, eq. 12.5)

$$NCov(p_{gh}, p_{ij}) = \delta_{gi}\delta_{hj}\pi_{gh}\pi_{ij} - \pi_{gh}\pi_{ij},$$

where

$$\delta_{ij} = \begin{cases} 0 & if \quad i \neq j \\ 1 & if \quad i = j \end{cases}$$

If the hypothesis of homogeneity holds, this statistic is distributed as χ^2 with $k-1$ degrees of freedom.

The homogeneity test is particularly useful in two-wave longitudinal studies to test the hypothesis that the distribution of a variable has not changed from the first occasion to the second.

To test homogeneity in PRELIS include the command

HT *varlist*

in the input file. PRELIS tests the homogeneity pairwise for every pair of variables in *varlist*. Note that this test can be applied to nominal as well as ordinal variables.

For ordinal variables, the homogeneity test (HT*est*) described here is different from the equal thresholds test (ET*test*) in two ways:

- It does not assume underlying normal variables.
- If underlying normality is assumed, the homogeneity hypothesis implies the equal thresholds hypothesis.

Example

Aish & Jöreskog (1990) analyze data on political attitudes. Their data consists of six ordinal variables measured on the same people at two occasions. The six variables are considered to be indicators of *Political Efficacy* and *System Responsiveness*. The input file **ex10a.prl** will read the 12 variables (six variables on two occasions) for every odd-numbered case and test the hypothesis that the univariate marginal distribution is stable over time for each of the six variables.

```
EXAMPLE 10A
TESTING HOMOGENEITY FOR EACH VARIABLE OVER TIME
POLITICAL ACTION PANEL DATA FOR USA
DA NI=12 MI=8,9
LA
NOSAY1 VOTING1 COMPLEX1 NOCARE1 TOUCH1 INTERES1
NOSAY2 VOTING2 COMPLEX2 NOCARE2 TOUCH2 INTERES2
RA=PANUSAF.RAW FO
(12f2.0)
SC CASE=ODD                    !This selects every odd-numbered case
CL ALL 1=AS 2=A 3=D 4=DS
HT NOSAY1 NOSAY2
HT VOTING1 VOTING2
HT COMPLEX1 COMPLEX2
HT NOCARE1 NOCARE2
HT TOUCH1 TOUCH2
HT INTERES1 INTERES2
OU MA=TM XB
```

Some matrix must be specified on the OU command to make the program compute all the bivariate marginal contingency tables that are needed for the calculation of the test statistics. Any matric appropriate for ordinal variable will do, that is, OM, PM, RM, and TM. Here we use MA = TM.

With ordinal variables, PRELIS gives a bivariate contingency table for each pair of variables, both in absolute frequencies and in percentages. With 12 variables, as in this example, there will be 66 such tables of each kind. One can put the XB option on the OU command to skip the printing of these tables in the output file.

```
         Homogeneity Tests

 Variable vs. Variable Chi-Squ. D.F. P-Value
   NOSAY1 vs.   NOSAY2   4.737    3    0.192
  VOTING1 vs.  VOTING2   2.494    3    0.476
 COMPLEX1 vs. COMPLEX2   8.767    3    0.033
  NOCARE1 vs.  NOCARE2   4.377    3    0.223
   TOUCH1 vs.   TOUCH2   8.087    3    0.044
 INTERES1 vs. INTERES2   4.450    3    0.217
```

None of the tests are significant at the one percent level, whereas the tests for COMPLEX and TOUCH are significant at the five percent level. Thus, it appears that the marginal distributions are not changing much over time.

References

Agresti, A. (1990) *Categorical Data Analysis*. John Wiley and Sons, New York.

Aish, A.M. & Jöreskog, K.G. (1990). A panel model for political efficacy and responsiveness: An application of LISREL 7 with weighted least squares. *Quality and Quantity*, **24**, 405-426.