Scientific Software International

Two-wave models

In this second example of analysis from longitudinal studies, we use LISREL to analyze data from Anderson & Maier (1963) and Hilton (1969).

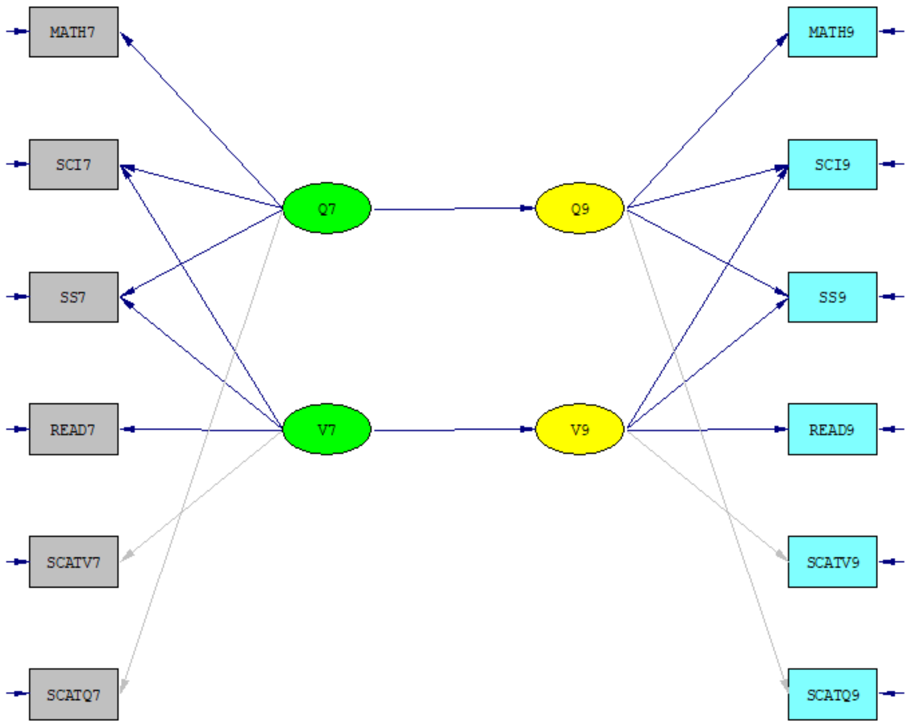
Educational Testing Service tested a nationwide sample of fifth graders in 1961, and then again in 1963, 1965, and 1967 as seventh, ninth, and eleventh graders, respectively. The test scores include the verbal (SCATV) and quantitative (SCATQ) parts of the Scholastic Aptitude Test (SCAT), and achievement tests in mathematics (MATH), science (SCI), social studies (SS), reading (READ), listening (LIST), and writing (WRIT). The examinees were divided into four groups according to gender and whether or not they participated in an academic curriculum in Grade 12. The four groups and their sample sizes are as follows:

- Boys academic (BA): $N = 373$
- Boys nonacademic (BNA): $N = 249$
- Girls academic (GA): $N = 383$
- Girls nonacademic (GNA): $N = 387$

Scores on each test have been scaled so that the unit of measurement is approximately the same at all occasions.

In this example we use the six tests MATH, SCI, SS, READ, SCATV, and SCATQ in Grades 7 and 9 only, and only for the group GA. Earlier studies (Jöreskog, 1970b) suggest that these tests measure two oblique factors that may reasonably be interpreted as a verbal (V) and a quantitative (Q) factor.

We set up the model shown below, which represents a model for the measurement of change in verbal and quantitative ability between Grades 7 and 9. Since there are no background variables in this model, we may for estimation purposes treat the pretests as the independent variables.



Note that the model includes the following features:

1. On each occasion the factor pattern is postulated to be restricted in the following way: MATH and SCATQ are pure measures of Q. READ and SCATV are pure measures of V. SCI and SS are composite measures of V and Q. This implies that there are four zero loadings in both $\mathbf{\Lambda}_y$ and $\mathbf{\Lambda}_x$. To fix the scales for V and Q, we assume that they are measured in the same units as SCATV and SCATQ, respectively. This means that there is a fixed 1 in each column of $\mathbf{\Lambda}_y$ and $\mathbf{\Lambda}_x$ and SCATV and SCATQ are reference variables.
2. It is postulated that Q_7 affects Q_9 only and not V_9, and similarly V_7. This means that there are two zero coefficients in $\mathbf{\Gamma}$. Furthermore, we postulate that the residuals ζ_1 and ζ_2 are uncorrelated, which means that, whatever remains in Q_9 and V_9 after Q_7 and V_7 are accounted for is uncorrelated with everything else.
3. The errors or unique factors in $\boldsymbol{\delta}$ and $\boldsymbol{\varepsilon}$ are assumed to be uncorrelated both within and between occasions.

The command file for this model is (**EX65A.LIS**) which is given in the **Lisrel examples\SEMs with latent variables** folder:

```
Verbal and Quantitative Ability In Grades 7 and 9.
Model: GA = DI and PS = DI
DA NI=12 NO=383
LA
MATH7 SCI7 SS7 READ7 SCATV7 SCATQ7 MATH9 SCI9 SS9 READ9 SCATV9 SCATQ9
KM FI=EX65.DAT
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```
SD FI=EX65.DAT
SE
7 8 9 10 11 12 1 2 3 4 5 6
MO NX=6 NY=6 NK=2 NE=2
LE
Q9 V9
LK
Q7 V7
FI GA 1 2 GA 2 1
FR LY 1 1 LY 2 1 LY 2 2 LY 3 1 LY 3 2 LY 4 2
FR LX 1 1 LX 2 1 LX 2 2 LX 3 1 LX 3 2 LX 4 2
VA 1 LX 5 2 LX 6 1
VA 1 LY 5 2 LY 6 1
OU ND=2 SE TV MI RS
```

The maximum likelihood estimates are given below. The rather low loadings of SCI and SS on Q at both occasions may seem a little surprising. However, and inspection of the items in tests SCI and SS reveals that these are mostly verbal problems concerned with logical reasoning, in contrast to the items in SCATQ, which are mostly numerical items measuring the ability to work with numbers. The small residual variance 1.85 of ς_2 means that V_9 can be predicted almost perfectly from V_7. This is not quite so for Q_7 since here we have a residual variance of 18.50. However, this may be due to the more rapid increase in variance of Q from Grade 7 to 9, which is manifested in the increase in variances, which is 143.55 – 103.87 = 39.68 for Q and 117.15 – 115.40 = 1.75 for V.

There is a reason not to look at each number in Table 6.7 too seriously, and this is the poor overall fit of the model as evidenced by the χ^2-value of 217.68 with 47 *df*. We shall therefore investigate the reason for this poor fit and demonstrate that LISREL may be used not only to assess or measure the goodness-of-fit of a model, but also to detect the parts of the model where the fit is poor. Taking the more fundamental assumptions of linearity and multinormality for granted, lack of fit of the model may be due to the fact that one or more of the postulates 1, 2, or 3 is not reasonable. We shall therefore investigate each of these separately.

$$\hat{\mathbf{\Lambda}}_X = \begin{bmatrix} 0.97 & 0 \\ 0.20 & 0.52 \\ 0.25 & 0.84 \\ 0 & 1.21 \\ 0 & 1.00 \\ 1.000 & 0 \end{bmatrix} \qquad \hat{\mathbf{\Theta}}_\delta = \begin{bmatrix} 32.25 \\ 30.14 \\ 43.63 \\ 46.19 \\ 19.69 \\ 50.40 \end{bmatrix}$$

$$\hat{\mathbf{\Lambda}}_Y = \begin{bmatrix} 0.89 & 0 \\ 0.24 & 0.64 \\ 0.36 & 0.69 \\ 0 & 0.95 \\ 0 & 1.00 \\ 1.00 & 0 \end{bmatrix} \qquad \hat{\mathbf{\Theta}}_\varepsilon = \begin{bmatrix} 23.07 \\ 43.20 \\ 52.46 \\ 42.15 \\ 19.69 \\ 50.40 \end{bmatrix}$$

$$\hat{\mathbf{\Gamma}} = \begin{bmatrix} 1.10 & 0 \\ 0 & 1.00 \end{bmatrix} \qquad \hat{\mathbf{\Phi}} = \begin{bmatrix} 103.87 & 92.85 \\ 92.85 & 115.40 \end{bmatrix}$$

$$\hat{\mathbf{\Psi}} = \begin{bmatrix} 18.50 & 0 \\ 0 & 1.85 \end{bmatrix} \qquad \hat{\mathbf{\Omega}} = \begin{bmatrix} 143.55 & 101.54 \\ 101.54 & 117.15 \end{bmatrix}$$

χ^2 = 217.68 with *df* = 47.

To investigate postulate 1, we set up a factor analysis of the pre- and post-tests separately, assuming the postulated two-factor structure. This gives χ^2 = 17.64 for the pretests and 2.62 for the post-tests, both with 10 degrees of freedom. Although the fit is not quite acceptable in Grade 7, we take the postulated factor structure to hold for both the pre- and post-tests. So we must continue to look for lack of fit due to postulate 2 or 3.

Postulate 2 is concerned with the interrelationships between the four factors Q_7, V_7 , Q_9, and V_9 . The most general assumption is that these four factors are freely intercorrelated, and this is equivalent to a LISREL model with all four coefficients in $\mathbf{\Gamma}$ free and with $\mathbf{\Psi}$ free as a full symmetric matrix. Hence, it is clear that the assumption made by 2. is the intersection of the two hypotheses "$\mathbf{\Gamma}$ is diagonal" and "$\mathbf{\Psi}$ is diagonal." It is therefore useful to test each of the four possible hypotheses. The results of these analyses may be presented in a 2 x 2 table as shown below. The row marginal of the table represent χ^2-values with one degree of freedom for testing the hypothesis that $\mathbf{\Psi}$ is diagonal. It is seen that this hypothesis should be rejected. The column marginals represent χ^2-values with *df* = 3 for testing the hypothesis that $\mathbf{\Gamma}$ is diagonal. This hypothesis seems quite reasonable. From these analyses it is clear that "$\mathbf{\Gamma}$ diagonal and $\mathbf{\Psi}$ free" is the most reasonable assumption to retain. The overall fit of this model is χ^2 = 196.2 with 46 *df*. Since this is still too large, we must continue to investigate postulate 3.

	$\mathbf{\Psi}$ diagonal	$\mathbf{\Psi}$ free	
$\mathbf{\Gamma}$ diagonal	χ^2_{47} =217.7	χ^2_{46} = 196.2	χ^2_{1} = 21.5
$\mathbf{\Gamma}$ free	χ^2_{45} = 216.7	χ^2_{44} = 193.6	χ^2_{1} = 23.1
	χ^2_{2} = 1.0	χ^2_{2} = 1.6	

The assumption in postulate 3 is that the unique factors in $\boldsymbol{\delta}$ and $\boldsymbol{\varepsilon}$ are uncorrelated both within and between sets. That they are uncorrelated within sets should not be questioned, since we have already found that the postulated factor-analysis model holds for both pre- and post-test. That they are uncorrelated between sets, however, is more questionable because of specific factors in each test. This means that the unique factors for corresponding tests should be allowed to correlated. To account for such correlations, Jöreskog (1970b) introduced so-called test-specific factors, that is, factors that do not contribute to correlations between tests within occasions but between the *same* tests at different occasions. In this case, there are only 2 occasions, it is not possible to define (identify) test-specific factors; we can merely introduce correlations between unique factors for corresponding pre- and post-tests.

All these analyses are in fact unnecessary as the output file from the initial model suggests immediately that the largest source of misspecification in the model is likely to be the autocorrelation, *i.e*, the correlation between error terms for the same variables over time. This can be seen in the section of the output called STANDARDIZED RESIDUALS.

The model previously given is now modified. This revised model can also be estimated with LISREL. In order to accommodate the correlated error terms we must write the model as a Submodel 3B with parameter matrices $\mathbf{\Lambda}_y$, $\mathbf{B}$, $\mathbf{\Psi}$ and $\mathbf{\Theta}_\varepsilon$.

The command file **EX65B.LIS** is:

```
Verbal and Quantitative Ability in Grades 7 and 9.
Model: BE diagonal and Autocorrelated errors
DA NI=12 NO=383
LA
MATH7 SCI7 SS7 READ7 SCATV7 SCATQ7 MATH9 SCI9 SS9 READ9 SCATV9 SCATQ9
KM FI=EX65.DAT
SD FI=EX65.DAT
MO NY=12 NE=4 BE=FU TE=SY PS=SY,FR
LE
Q7 V7 Q9 V9
FR LY 1 1 LY 2 1 LY 2 2 LY 3 1 LY 3 2 LY 4 2
FR LY 7 3 LY 8 3 LY 8 4 LY 9 3 LY 9 4 LY 10 4
VA 1 LY 5 2 LY 6 1
VA 1 LY 11 4 LY 12 3
FR BE 3 1 BE 4 2
FI PS 3 1 PS 3 2 PS 4 1 PS 4 2
```

```
VA 0.0 PS 3 1 PS 3 2 PS 4 1 PS 4 2
FR TE 7 1 TE 8 2 TE 9 3 TE 10 4 TE 11 5 TE 12 6
PD
OU ME=ML AD=OFF NS SE TV RS
```

The analysis of the revised model gives results shown below. All estimated parameters are significantly different from zero.

$$\hat{\mathbf{\Lambda}}_Y = \begin{bmatrix} 1.01 & 0 & 0 & 0 \\ 0.13 & 0.60 & 0 & 0 \\ 0.12 & 0.98 & 0 & 0 \\ 0 & 1.24 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0.93 & 0 \\ 0 & 0 & 0.13 & 0.77 \\ 0 & 0 & 0.25 & 0.82 \\ 0 & 0 & 0 & 0.98 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{\Theta}_\varepsilon = \begin{bmatrix} 27.75 \\ 29.59 \\ 40.28 \\ 44.21 \\ 24.37 \\ 54.24 \\ 17.66 \\ 41.14 \\ 50.90 \\ 40.34 \\ 24.84 \\ 74.52 \end{bmatrix}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.06 & 0 & 0 & 0 \\ 0 & 0.98 & 0 & 0 \end{bmatrix} \qquad \hat{\mathbf{\Psi}} = \begin{bmatrix} 100.57 & & & \\ 90.53 & 110.45 & & \\ 0 & 0 & 22.63 & \\ 0 & 0 & 8.42 & 6.94 \end{bmatrix}$$

χ^2 = 65.61 with *df* = 40.

i	$Var(\varepsilon_i)$	$Var(\varepsilon_{i+6})$	$Cov(\varepsilon_i \varepsilon_{i+6})$	$Corr(\varepsilon_i \varepsilon_{i+6})$
1	27.75 (3.78)	17.66 (3.88)	-3.47 (2.78)	-0.157
2	29.59 (2.37)	41.14 (3.38)	9.60 (2.10)	0.275
3	40.28 (3.60)	50.90 (4.21)	6.16 (2.82)	0.136
4	44.21 (4.25)	40.34 (3.59)	7.52 (2.89)	0.178
5	24.37 (2.47)	24.84 (2.60)	12.04 (2.05)	0.489
6	54.24 (4.87)	74.52 (6.73)	22.83 (4.40)	0.359

The test of overall goodness of fit gives χ^2 = 65.61 with 40 *df*. This represents a reasonably good fit of the model to the data. An approximate test of the hypothesis that the unique factors are uncorrelated between occasions is obtained as χ^2 = 196.2 – 65.5 = 130.6 with 6 *df*, thus this hypothesis is quite unreasonable. The variances, covariances, and correlations of the unique factors are shown above. A comparison of the covariances with their standard errors reveals that all covariances, except possibly the one between ε_1 and ε_7 , are significantly non-zero.