

Longitudinal ordinal data

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1. Introduction

This example is the first of a set of examples extracted from a note by K.G. Jöreskog first posted on the SSI website in 2005 with the title “*Structural Equation Modeling with Ordinal Variables using LISREL*”.

In the two previous examples I explained how one can analyze ordinal variables in cross-sectional studies.

This section considers models for analyzing data from longitudinal studies where the same individuals are observed or measured at two or more occasions. Such studies are very common in psychology, education, sociology and other fields. Many papers have considered the specification of models incorporating causation and measurement errors in the analysis of data from panel studies, and statistical models and methods for analysis of longitudinal data.

The characteristic feature of a longitudinal research design is that the same measurement instruments are used on the same people at two or more occasions. The purpose of a longitudinal or panel study is to assess the changes that occur between the occasions, and to attribute these changes to certain background characteristics and events existing or occurring before the first occasion and/or to various treatments and developments that occur after the first occasion. Often, when the same variables are used repeatedly, there is a tendency for the measurement errors in these variables to correlate over time because of specific factors, memory or other retest effects. Hence there is a need to consider models with correlated measurement errors.

The analysis of ordinal variables in longitudinal studies requires special techniques and procedures which are different from those used with continuous longitudinal variables. This section illustrates these techniques and procedures using the Political Action Panel Study for the USA which is a two-wave panel study. The model considered here is an extension of a model developed by Aish & Jöreskog (1990). The original USA sample consisted of 1719 cases interviewed in 1974. Five years later 933 of these cases were reinterviewed using the same six political efficacy items that were analyzed in Section 2.

In order to estimate differences in means and variances of latent variables over time one must ascertain that the latent variables are on the same scale at different occasions. Both the origin and the unit of measurement must be the same over time. If the observed indicators are continuous, this can be achieved by anchoring each latent variable in one of its observed indicators, a so-called reference variable, and by assuming that the mean of the latent variable is zero at one occasion, e.g., the first. By choosing the same reference variable at all occasions one can ascertain that this latent variable is on the same scale over time. However, if the observed indicators are ordinal this is not sufficient, for ordinal variables do not have metric scales so it is meaningless to say that they are on the same scale over time, see the second example in this set of examples. Again we must use the underlying variables instead of the observed ordinal variables. The underlying variables can be put on the same scale by assuming equal thresholds for the underlying variables of the same ordinal variable across time.

2. Equal Thresholds

Consider k ordinal variables z_1, z_2, \dots, z_k , with m_1, m_2, \dots, m_k , categories, respectively. Assume that these ordinal variables have been measured on N individuals at T occasions, as in a longitudinal study. Denote by $\pi_{it,c}$ the probability of a response in category c of variable i at time t , $c = 1, 2, \dots, m_i$, $i = 1, 2, \dots, k$, $t = 1, 2, \dots, T$. Let

$$\tau_{i,1} < \tau_{i,2} < \dots < \tau_{i,m_i-1}$$

be a set of thresholds for variable i assumed to be the same at all occasions. Assuming that the underlying variable z_{it}^* is normally distributed with mean μ_{it} and standard deviation σ_{it} at time t , the probability of a response in category c of variable i at time t is

$$\pi_{it,c} = \int_{(\tau_{i,c-1} - \mu_{it})/\sigma_{it}}^{(\tau_{i,c} - \mu_{it})/\sigma_{it}} \phi(u) du, \quad (11)$$

where ϕ is the standard normal density function. Consider a given variable i with m categories. Omitting index i , equation (11) becomes

$$\pi_{t,c} = \int_{(\tau_{c-1} - \mu_t)/\sigma_{it}}^{(\tau_c - \mu_t)/\sigma_{it}} \phi(u) du, \quad (12)$$

The probability of a response in category a or lower is

$$\pi_{t,a}^* = \sum_{c=1}^a \pi_{t,c} = \int_{-\infty}^{(\tau_a - \mu_t)/\sigma_t} \phi(u) du = \Phi\left(\frac{\tau_a - \mu_t}{\sigma_t}\right), \quad (13)$$

where Φ is the distribution function of the standard normal distribution. There are two fundamental indeterminacies in (13). One can add a constant to all the τ 's and to μ and one can multiply all the τ 's, μ 's, and σ by a non-zero constant without altering the right hand side of the equation. In other words, a linear transformation of z^* changes the τ 's, μ 's, and σ while retaining normality and leaving (13) unchanged. This is a reflection of the fact that although the scale of z^* is the same over time, the origin and unit of measurement are still arbitrary. Solving for $\tau_a - \mu_t / \sigma_t$, equation (13) can be written

$$\frac{\tau_a - \mu_t}{\sigma_t} = \Phi^{-1}(\pi_{a,t}^*), \quad (14)$$

or, equivalently,

$$\tau_a = \mu_t + \sigma_t \Phi^{-1}(\pi_{a,t}^*) = \mu_t + \sigma_t \tau_{a,t}^*, \quad (15)$$

where Φ^{-1} is the inverse function of Φ . The quantity $\Phi^{-1}(\pi_{a,t}^*)$ on the right side of (14) and in the middle of (15) is the unconstrained threshold $\tau_{a,t}^*$ determined for each variable from the univariate marginal probabilities at each time. It is convenient to refer to the τ_a on the left side of equation (15) as the constrained threshold. Equation (15) represents a set of constraints on μ_t and σ_t because the right hand side varies with t whereas the left hand side does not. If $m > 3$, the common thresholds, μ_t and σ_t can be estimated from the univariate marginal data of those variables whose thresholds are supposed to be equal. If $m = 2$, i.e., if a variable is dichotomous, only one of μ_t or σ_t can be estimated. In this case, PRELIS sets $\sigma_t = 1$ and estimates μ_t .

To identify the parameters, the origin and the unit of measurement of the common scale must be fixed. In the standard parameterization, this is done such that $\sum_{t=1}^T \hat{\mu}_t = 0$ and $\sum_{t=1}^T \hat{\sigma}_t^2 = T$, i.e., the average mean is 0 and the average variance is 1. In the alternative parameterization, the scale is fixed by $\tau_1 = 0$ and $\tau_2 = 1$.

For $T = 2$ and the standard parameterization, equal thresholds is illustrated in the following table

Variable	Mean	Std. Dev.	Thresholds				
1	μ_1	σ_1	τ_1	τ_2	τ_3	\cdots	τ_{m-1}
2	μ_2	σ_2	τ_1	τ_2	τ_3	\cdots	τ_{m-1}

where $\mu_1 + \mu_2 = 0$ and $\sigma_1^2 + \sigma_2^2 = 2$.

For $T = 2$ and the alternative parameterization, equal thresholds is illustrated in the following table

Variable	Mean	Std. Dev.	Thresholds				
1	μ_1^*	σ_1^*	0	1	τ_3^*	...	τ_{m-1}^*
2	μ_2^*	σ_2^*	0	1	τ_3^*	...	τ_{m-1}^*

The alternative parameterization is obtained from the standard parameterization as follows.

1. For each variable, subtract τ_1 from all the thresholds and from μ .
2. For each variable, divide all thresholds and μ and σ by $\tau_2 - \tau_1$.

The estimated means and standard deviations of the underlying variables can be used to form an estimated mean vector and covariance matrix for all variables. These matrices can be used in LISREL in the usual way. The procedures are illustrated in the following sections.

3. A Two Variables Example of Equal Thresholds

First, I will illustrate equal thresholds by means of the two variables data that I introduced in the previous example in this set. It consists of 5000 cases on two variables on a five-point scale. To estimate the thresholds and the polychoric correlation under the condition of equal thresholds and the standard parameterization, use the following PRELIS command file (file **TWOVARS.PRL**):

```
DA NI=3
RA=TWOVARS.FRQ
LA
FREQ X1 X2
WEIGHT FREQ
ET X1 X2
OU MA=PM
```

This gives the following marginal parameters

Univariate Marginal Parameters

Variable	Mean	St. Dev.	Thresholds			
-----	-----	-----	-----	-----	-----	-----
X1	-0.498	1.003	-1.304	-0.290	0.388	1.492
X2	0.498	0.997	-1.304	-0.290	0.388	1.492

Note that

- The thresholds are equal
- The sum of the means is 0
- The sum of squares of the standard deviations is 2

To estimate the marginal parameters under the alternative parameterization one can either put AP on the OU line or replace MA = PM by MA = CM, see file **TWOVARS32.PRL**. This gives the following univariate marginal parameters

Univariate Marginal Parameters

Variable	Mean	St. Dev.	Thresholds			
-----	-----	-----	-----	-----	-----	-----
X1	0.795	0.989	0.000	1.000	1.669	2.757
X2	1.777	0.983	0.000	1.000	1.669	2.757

The thresholds are still equal but the second and third property no longer holds. The underlying variables are now on a different scale.

To obtain a test of the hypothesis of equal thresholds, one can run **TWOVARS31.PRL** with and without the ET line and compute the difference in chi-square for test of underlying bivariate normality. With ET this gives a chi-square of 18.411 with 19 degrees of freedom. Without ET this gives a chi-square of 17.249 with 15 degrees of freedom. The test of equal thresholds gives a chi-square of $18.411 - 17.249 = 1.162$ with $19 - 15 = 4$ degrees of freedom. Hence, the hypothesis of equal thresholds cannot be rejected.

4 Estimating the Mean Vector and Covariance Matrix under Equal Thresholds

Most of the remaining part of this section will be devoted to the analysis of the USA panel data of the political efficacy items. The six political efficacy items NOSAY, VOTING, COMPLEX, NOCARE, TOUCH, and INTEREST and their response category coding were introduced in the second example. The difference now is that I will be analyzing the panel data consisting of the 933 cases answering the same items at two points in time with a five-year time lag. The panel data in free format is available in the file **PANUSA.RAW**. There are 12 variables. The first six are responses to the efficacy items at time 1 and the second are responses to the same items at time 2.

The PRELIS command file below (**ORD31.PRL**) will do the following

Eliminate the VOTING item since this will not be used, see the second example for the reason for this.

- Impute missing values of the time 2 variables using time 1 variables as matching variables (for information on imputation, see the first example).
- Estimate the mean vector and the covariance matrix of the underlying variables under the condition of equal threshold for each item over time.
- The asymptotic covariance matrix of the covariance matrix will also be estimated.
- The mean vector, the covariance matrix, and the asymptotic covariance matrix are saved in files.

```
PANELUSA: PRELIS Run1
Estimating Mean Vector and Covariance Matrix under Equal Tresholds
DA NI=12 MI=8,9
LA
NOSAY1 VOTING1 COMPLEX1 NOCARE1 TOUCH1 INTERES1
NOSAY2 VOTING2 COMPLEX2 NOCARE2 TOUCH2 INTERES2
RA=PANUSA.RAW
SD VOTING1 VOTING2
```

```

IM (NOSAY2 - INTERES2) (NOSAY1 - INTERES2) XN
ET NOSAY1 NOSAY2
ET COMPLEX1 COMPLEX2
ET NOCARE1 NOCARE2
ET TOUCH1 TOUCH2
ET INTERES1 INTERES2
Output MA=CM ME=PANUSA.ME CM=PANUSA.CM AC=PANUSA.ACC

```

After imputation there are 849 cases with complete data on all 10 variables. The univariate marginal parameters are estimated under the alternative parameterization as

Univariate Marginal Parameters

Variable	Mean	St. Dev.	Thresholds		
-----	-----	-----	-----	-----	-----
NOSAY1	1.289	0.959	0.000	1.000	2.675
COMPLEX1	0.579	0.627	0.000	1.000	1.676
NOCARE1	0.926	0.793	0.000	1.000	2.359
TOUCH1	0.705	0.679	0.000	1.000	2.174
INTERES1	0.817	0.761	0.000	1.000	2.399
NOSAY2	1.398	0.837	0.000	1.000	2.675
COMPLEX2	0.635	0.551	0.000	1.000	1.676
NOCARE2	0.998	0.675	0.000	1.000	2.359
TOUCH2	0.665	0.579	0.000	1.000	2.174
INTERES2	0.849	0.644	0.000	1.000	2.399

Note that the thresholds are equal for each item over time. The covariance matrix of the 10 variables is estimated as

Covariance Matrix

	NOSAY1	COMPLEX1	NOCARE1	TOUCH1	INTERES1	NOSAY2
-----	-----	-----	-----	-----	-----	-----
NOSAY1	0.920					
COMPLEX1	0.238	0.393				
NOCARE1	0.456	0.210	0.628			
TOUCH1	0.298	0.141	0.341	0.461		
INTERES1	0.367	0.175	0.408	0.368	0.579	
NOSAY2	0.341	0.114	0.236	0.133	0.195	0.701
COMPLEX2	0.140	0.156	0.098	0.061	0.097	0.167
NOCARE2	0.270	0.111	0.256	0.169	0.235	0.338
TOUCH2	0.160	0.071	0.141	0.141	0.183	0.185
INTERES2	0.224	0.105	0.207	0.155	0.240	0.245

Covariance Matrix

	COMPLEX2	NOCARE2	TOUCH2	INTERES2
-----	-----	-----	-----	-----
COMPLEX2	0.304			
NOCARE2	0.140	0.456		
TOUCH2	0.069	0.250	0.336	
INTERES2	0.104	0.312	0.264	0.415

By this procedure the mean vector and covariance matrix have been estimated such that the underlying variables of each item are on the same scale across time, at least to a sufficient degree of approximation. One can therefore proceed as if these underlying variables had been measured on the same scale over time. This assumption is fundamental for the rest of the analysis.

5. A Panel Model for Efficacy

In this section I develop a panel model for efficacy. The measurement model involves two components of efficacy called *Efficacy* and *Respons*. It is the same as considered in the second example and estimated from cross-sectional data. This measurement model is applied at each time point.

The objective of the panel model is to answer such questions as: Has the level of efficacy increased or decreased over time? Has the variance of efficacy increased or decreased over time?

A conceptual path diagram of the panel model for efficacy is shown in Fig. 4.

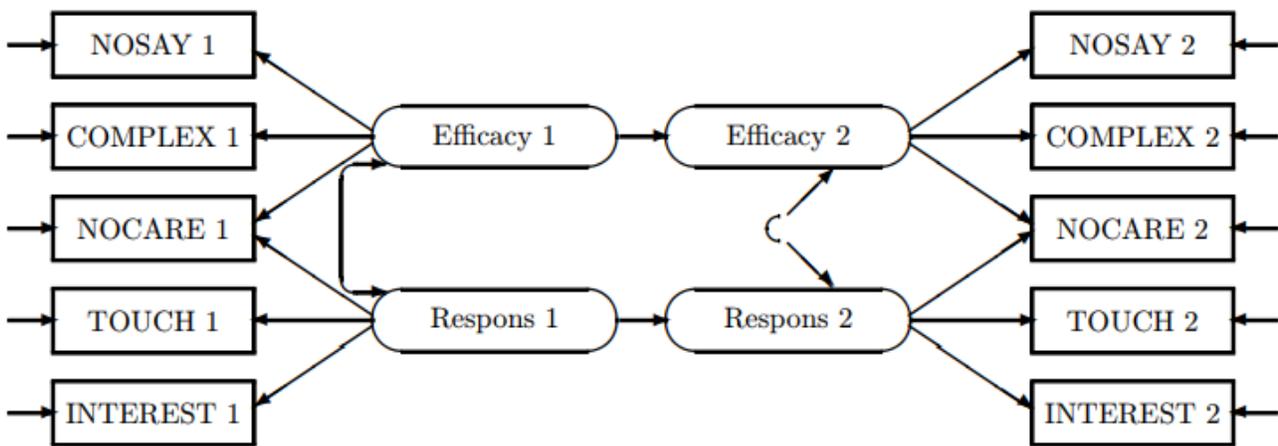


Figure 4: Panel Model for Efficacy and Respons

In the following I refer to the observed variables in the sense of their underlying variables. Thus, when I refer to COMPLEX1 and COMPLEX2, for example, I mean the variables underlying COMPLEX1 and COMPLEX2.

The model also involves a structural model in the middle of the path diagram in which Efficacy at time 2 is predicted by *Efficacy* at time 1 without the use of *Respons* at time 1, and *Respons* at time 2 is predicted by *Respons* at time 1 without the use of *Efficacy* at time 1. In addition to these features, the model includes several features not visible in Fig. 4.

The measurement error in each variable at time 1 correlates with the measurement error in the corresponding variable at time 2 due to a specific factor in each item. To explain this further, I take COMPLEX as an example. Let x be COMPLEX1 and y be COMPLEX2. Then the measurement equations for COMPLEX1 and COMPLEX2 can be written

$$\text{Time 1: } x = \lambda_1 \xi + \delta = \lambda_1 \xi + s + d$$

$$\text{Time 2: } y = \lambda_2 \eta + \varepsilon = \lambda_2 \eta + s + e,$$

where ξ is Efficac1 and η is Efficac2, δ and ε are the so called measurement errors in the LISREL model. Each of these error terms are the sum of two components, one specific factor s unique to the item COMPLEX, and one pure random error component, d and e , respectively, where d and e are uncorrelated. It follows that δ and ε are correlated and that

$$\text{Cov}(\delta, \varepsilon) = \text{Var}(s).$$

Thus, the specific error variance can be estimated as the autocovariance between the LISREL measurement errors.

- The loading of NOSAY1 on Efficac1 and of NOSAY2 on Efficac2 are fixed to 1 to fix the unit of measurement for Efficac1 and Efficac2. Since NOSAY1 and NOSAY2 have the same unit of measurement (by the construction of equal thresholds), Efficac1 and Efficac2 will also have the same unit of measurement. Similarly, the loadings of INTERES1 on *Respons1* and of INTERES2 on *Respons2* are fixed to 1 to fix the unit of measurement for *Respons1* and *Respons2*. Since INTERES1 and INTERES2 have the same unit of measurement, *Respons1* and *Respons2* will also have the same unit of measurement.
- The other four loadings on the latent variables are constrained to be the same across time.
- There is also an intercept term (not visible in the path diagram) in each measurement equation. These intercept terms are also constrained to be equal across time.
- The equality of intercepts and factor loadings across time is necessary in order to compare the latent variables over time on the same scale, i.e., with the same origin and unit of measurement.

5.1.1 Input

A SIMPLIS command file for estimating this panel model is (file **ORD32.SPL**):

```
SIMPLIS File for Estimating the Panel Model
Observed Variables:                ! 1
NOSAY1 COMPLEX1 NOCARE1 TOUCH1 INTERES1 ! 2
NOSAY2 COMPLEX2 NOCARE2 TOUCH2 INTERES2 ! 3
Means from File PANUSA.ME          ! 4
Covariance Matrix from File PANUSA.CM ! 5
Asymptotic Covariance Matrix from File PANUSA.ACC ! 6
Sample Size: 849                   ! 7
Latent Variables: Efficac1 Respons1 Efficac2 Respons2 ! 8
Relationships                       ! 9

NOSAY1 - NOCARE1 = CONST Efficac1 ! 10
NOCARE1 - INTERES1 = CONST Respons1 ! 11
NOSAY1 = 1*Efficac1                ! 12
INTERES1 = 1*Respons1              ! 13

NOSAY2 - NOCARE2 = CONST Efficac2 ! 14
NOCARE2 - INTERES2 = CONST Respons2 ! 15
NOSAY2 = 1*Efficac2                ! 16
```

```

INTERES2 = 1*Respons2                                ! 17

Let the errors of NOSAY1 and NOSAY2 correlate        ! 18
Let the errors of COMPLEX1 and COMPLEX2 correlate   ! 19
Let the errors of NOCARE1 and NOCARE2 correlate     ! 20
Let the errors of TOUCH1 and TOUCH2 correlate       ! 21
Let the errors of INTERES1 and INTERES2 correlate   ! 22

Set Efficac1 -> COMPLEX1 = Efficac2 -> COMPLEX2     ! 23
Set Efficac1 -> NOCARE1 = Efficac2 -> NOCARE2       ! 24
Set Respons1 -> NOCARE1 = Respons2 -> NOCARE2       ! 25
Set Respons1 -> TOUCH1 = Respons2 -> TOUCH2        ! 26

Set CONST -> NOSAY1 = CONST -> NOSAY2              ! 27
Set CONST -> COMPLEX1 = CONST -> COMPLEX2          ! 28
Set CONST -> NOCARE1 = CONST -> NOCARE2            ! 29
Set CONST -> TOUCH1 = CONST -> TOUCH2             ! 30
Set CONST -> INTERES1 = CONST -> INTERES2         ! 31

Efficac2 = CONST Efficac1                            ! 32
Respons2 = CONST Respons1                            ! 33

Let the errors of Efficac2 and Respons2 correlate   ! 34

Method of Estimation: Weighted Least Squares
Path Diagram
End of Problem

```

To refer to different lines in this input file, I have numbered the lines in the right margin.

Lines 1-7 specify the names of variables and the data. One can replace all these lines by the single line (see file **ORD32A.SPL**):

```
System File from File ORD31.DSF
```

The system file **ORD31.DSF**, obtained by running **ORD31.PRL**, has all the information about the variables and the data, even the location of the asymptotic covariance matrix.

The variable **CONST** is a variable which is equal to 1 for every case. This variable is always available in **SIMPLIS**; it need not be in the data. It is used to estimate an intercept term or a mean. For example,

```
Y = CONST X
```

is used to specify the regression of Y on X:

$$Y = \alpha + \gamma X.$$

α is the coefficient of **CONST** just like is the coefficient of X. One can also use **CONST** to estimate a mean. For example,

```
Y = CONST
```

will estimate the mean of Y as the coefficient of CONST. Y and X can be any variables, observed or latent. For further examples of CONST, see Jöreskog & Sörbom (1999b), Chapter 2.

Lines 10-11 give the measurement model at time 1 and lines 12-13 specify NOSAY1 and INTERES1 as reference variables for *Efficac1* and *Respons1*, respectively. Similarly lines 14-15 give the measurement model at time 2 and lines 16-17 specify the corresponding reference variables at time 2. Note that all the measurement equations include intercept terms. Also note that NOCARE loads on both *Efficacy* and *Respons*.

Lines 18-22 specify the autocorrelated measurement errors needed to estimate the specific factor in each item.

Lines 23-26 constrain the factor loadings to be the same at time 1 and time 2. Here *Efficac1* → COMPLEX1, for example, is short for "the path from *Efficac1* to COMPLEX1." There are 4 factor loadings that are supposed to be equal. The other two loadings at each time point are fixed at 1.

Lines 27-31 constrain the intercepts in the measurement equations to be equal at time 1 and time 2.

Lines 32-33 give the structural equations. The idea is that *Efficacy* at time 2 is predicted by *Efficacy* at time 1 without the use of *Respons* at time 1, and *Respons* at time 2 is predicted by *Respons* at time 1 without the use of *Efficacy* at time 1. These structural equations also include intercept terms. As will be seen later, these can be interpreted as the mean difference in the latent variables.

Line 34 specifies that the two error terms in the structural equations are allowed to be correlated. This is my way of saying that I do not think that the correlation between *Efficacy* and *Respons* at time 2 can be entirely explained by *Efficacy* and *Respons* at time 1. There may be many variables out there in the world that influences *Efficacy* and *Respons*. One such variable might be interest in politics, for example. If there is such a variable that affects all four latent variables but is not included in the model it implies that *Efficacy* and *Respons* will be correlated at time 2 even after controlling for the effects of *Efficacy* and *Respons* at time 1.

5.1.2 Output

The output from **ORD32.SPL** or **ORD32A.SPL** gives the following fit statistics

Degrees of Freedom = 31

Minimum Fit Function Chi-Square = 38.11 (P = 0.178)

indicating that the model fits quite well. The model was fitted by WLS using the inverse of the asymptotic covariance matrix as a weight matrix. The chi-square is a C1 in the sense of Jöreskog, et al. (2001), Chapter 4 and Appendix A. As explained in Section 2, one can also use ML to fit the model and the asymptotic covariance matrix to correct the chi-square for non-normality, see file **ORD32B.SPL**. This gives a C3 as

Degrees of Freedom for (C1)-(C3),C(5)	31
Satorra-Bentler (1988) Scaled Chi-Square (C3)	26.893 (P = 0.6775)

Note that these two ways of fitting the model gives approximately the same chi-square.

In the following I present results from the output of **ORD32.SPL** or **ORD32A.SPL**. The measurement equations are estimated as

Measurement Equations

NOSAY2 = 1.283 + 1.000*Efficac2, Errorvar.= 0.236 , R² = 0.600
 Standerr (0.0443) (0.0418)
 Z-values 28.975 5.651
 P-values 0.000 0.000

COMPLEX2 = 0.597 + 0.444*Efficac2, Errorvar.= 0.235 , R² = 0.229
 Standerr (0.0415) (0.0344) (0.0142)
 Z-values 14.397 12.919 16.514
 P-values 0.000 0.000 0.000

NOCARE2 = 0.932 + 0.437*Efficac2 + 0.581*Respons2, Errorvar.= 0.107 , R² = 0.731
 Standerr (0.0511) (0.0883) (0.0832) (0.0191)
 Z-values 18.222 4.949 6.981 5.582
 P-values 0.000 0.000 0.000 0.000

TOUCH2 = 0.687 + 0.797*Respons2, Errorvar.= 0.119 , R² = 0.635
 Standerr (0.0572) (0.0267) (0.0157)
 Z-values 11.999 29.858 7.590
 P-values 0.000 0.000 0.000

INTERES2 = 0.828 + 1.000*Respons2, Errorvar.= 0.0731 , R² = 0.817
 Standerr (0.0592) (0.0209)
 Z-values 13.990 3.507
 P-values 0.000 0.000

NOSAY1 = 1.283 + 1.000*Efficac1, Errorvar.= 0.377 , R² = 0.591
 Standerr (0.0443) (0.0628)
 Z-values 28.975 5.998
 P-values 0.000 0.000

COMPLEX1 = 0.597 + 0.444*Efficac1, Errorvar.= 0.286 , R² = 0.273
 Standerr (0.0415) (0.0344) (0.0195)
 Z-values 14.397 12.919 14.636
 P-values 0.000 0.000 0.000

NOCARE1 = 0.932 + 0.437*Efficac1 + 0.581*Respons1, Errorvar.= 0.168 , R² = 0.733
 Standerr (0.0511) (0.0883) (0.0832) (0.0294)
 Z-values 18.222 4.949 6.981 5.717
 P-values 0.000 0.000 0.000 0.000

TOUCH1 = 0.687 + 0.797*Respons1, Errorvar.= 0.171 , R² = 0.630
 Standerr (0.0572) (0.0267) (0.0224)
 Z-values 11.999 29.858 7.629
 P-values 0.000 0.000 0.000

INTERES1 = 0.828 + 1.000*Respons1, Errorvar.= 0.122 , R² = 0.790
 Standerr (0.0592) (0.0289)
 Z-values 13.990 4.206
 P-values 0.000 0.000

It is seen the measurement model is the same at both time points and that all parameters are statistically significant.

The error covariances (specific factors) are estimated as

```

Error Covariance for NOSAY1 and:NOSAY2 = 0.0450
                                         (0.0299)
                                         1.503

Error Covariance for COMPLEX1 and:COMPLEX2 = 0.104
                                         (0.0130)
                                         7.979

Error Covariance for NOCARE1 and:NOCARE2 = 0.0184
                                         (0.0120)
                                         1.532

Error Covariance for TOUCH1 and:TOUCH2 = 0.0138
                                         (0.0103)
                                         1.335

Error Covariance for INTERES1 and:INTERES2 = 0.0129
                                         (0.0116)
                                         1.104

```

All the estimates of error covariances are positive which is in line with the interpretation of them as variances of the specific factors. However, only the specific error variance of COMPLEX is statistically significant. This does not mean that the other specific factors do not exist, only that they are smaller than that of COMPLEX and that the sample is not large enough to make them significant.

The structural equations are estimated as

```

Efficac2 = 0.0411 + 0.497*Efficac1, Errorvar.= 0.219 , R² = 0.380
Standerr (0.0684) (0.0374) (0.0301)
Z-values 0.601 13.273 7.291
P-values 0.548 0.000 0.000

Respons2 = - 0.00557 + 0.490*Respons1, Errorvar.= 0.216 , R² = 0.337
Standerr (0.0579) (0.0305) (0.0173)
Z-values -0.0961 16.052 12.458
P-values 0.923 0.000 0.000

```

This shows that the stability coefficients are statistically significant suggesting that the latent variables at time 2 can be predicted from those of time 1, to some extent. But as indicated by the R^2 's the predictions are not very accurate. Other variables outside of the model may be needed to make these predictions more accurate. The covariance between the two error terms is estimated as

```

Error Covariance for Respons2 and:Efficac2 = 0.125
                                         (0.0160)
                                         7.828

```

indicating that this is highly significant.

The means of the latent variables cannot be determined on an absolute scale. Although *Efficac1* and *Efficac2* are on the same scale, the origin of the scale is undetermined. We can only estimate the mean difference between *Efficac2* and *Efficac1* and between *Respons2* and *Respons1*. We can fix the origin of the *Efficacy* scale at the mean of *Efficac1* and then estimate the mean of *Efficac2*. Similarly, we can fix the origin of the *Respons* scale at the mean of *Respons1* and estimate the mean of *Respons2*. By this convention (or identification condition) the mean differences equal the intercept terms in the structural equations. It is seen that both of these are non-significant indicating no change in level of the two latent variables over time. A larger sample is needed to be able to tell whether there is a change in level over time. If the intercept terms had been significant there is probably an increase rather than a decrease over time.

Further information about the four latent variables in the output is summarized in Tables 4 and 5.

Table 4: Estimated Means and Covariance Matrix for Efficacy

	Efficacy1	Efficacy2	Means
Efficacy1	0.55		0.00
Efficacy2	0.28	0.36	0.05

Table 5: Estimated Means and Covariance Matrix for Respons

	Respons1	Respons2	Means
Respons1	0.46		0.00
Respons2	0.22	0.33	-0.01

The variance of both *Efficacy* and *Respons* has decreased over time indicating that the population is more homogenous at time 2 than at time 1. The question is whether this decrease is statistically significant. It is not possible to test the hypothesis of equal variances of the latent variables over time using this model. One needs to use another parameterization for this. I will consider this in Section 4.4.

5.1.3 Testing Sequence

Some readers may say: How can I put up this model directly without first testing all the assumptions it is based on? Obviously, since the model fits the data well and all results make good sense, there is no need to test all the intermediate steps. However, in general this is a good idea. For instructional purposes, I will therefore go over the steps here. I leave it to the reader to actually do these steps.

There are several ways to do the tests. I recommend the following sequential testing procedure.

1. Test the measurement model at each time point separately. To test the measurement model at time 1, include only the lines 10-13. To test the measurement model at time 2, include only the lines 14-17. Selection of variables is automatic in SIMPLIS; only the variables included in the model will be used. If the measurement model fits at both occasions one can continue with the next step. If the measurement model fits at one occasion but not at the other occasion it means that the measurement models are not functionally equivalent, i.e., the observed variables do not measure the same latent variables at each

occasion. Then further analysis is meaningless. If the measurement models fail to fit at both occasions, both models should be modified and their functional equivalence should be tested again.

2. Test the two measurement models jointly. This is done by including only the lines 10-17. The covariance matrix of the four latent variables is left unconstrained. If this model does not fit (which is the case here), one must introduce the specific factors by adding the lines 18-22. If, after adding the specific factors, the model still does not fit, there must be something fundamentally wrong. This should not happen if Step 1 has been satisfactorily resolved.
3. Test the equality of factor loadings over time. This is done by adding the lines 23-26. The test of equality of factor loadings is obtained by computing the difference in chi-squares between this model and the previous one. If the hypothesis of equal factor loadings is rejected it will not be possible to compare the latent variables over time with the same unit of measurement (Under partial invariance of factor loadings, i.e., if some factor loadings are equal while others are different, such a comparison may still be possible, but this is very tricky.)
4. Test the equality of intercept terms over time. This is done by adding lines 27-31. The test of equality of intercept terms is obtained by computing the difference in chi-squares between this model and the previous one. If the hypothesis of equal intercept terms is rejected it will not be possible to compare the latent variables over time with the same origin (see note above at 3.)
5. Test the structural model. This is done by adding lines 32-34. This test is obtained by computing the difference in chi-squares between this model and the previous one. If the structural model does not fit, replace the two lines

```
Effiac2 = CONST Effiac1 !      32
Respons2 = CONST Respons1 !    33
```

with

```
Effiac2 = CONST Effiac1 Respons1 Respons2 = CONST Effiac1 Respons1
```

This model is equivalent to the model in Step 4. So it will fit if that model fits.

5.1.4 Testing Equality of Variances

In Section 2 I found that the factor variances decreased over time but I was unable to test whether this decrease is statistically significant. In this section I show how this can be done by a slight modification of the input.

Replace the lines

```
Effiac2 = CONST Effiac1      !      32
Respons2 = CONST Respons1    !      33
Let the errors of Effiac2 and Respons2 correlate !      34
```

with (see file **ORD33.SPL**)

```
Effiac1 = 0*CONST
Respons1 = 0*CONST
Effiac2 = CONST
Respons2 = CONST
```

This will leave the covariance matrix of the four latent variables unconstrained but the model is otherwise the same as before. In fact, the model is the same as in Step 4 of the previous section. This model has a chi-square of 34.369 with 29 degrees of freedom.

A test of equality of factor variances can now be obtained by adding the lines (see **ORD33A.SPL**):

```
Equal Variances: Efficac1 Efficac2  
Equal Variances: Respons1 Respons2
```

This model has a chi-square of 78.582 with 31 degrees of freedom. The difference in chi-squares is 44.2 with 2 degrees of freedom which is highly significant. The hypothesis of equal variances must be rejected. Thus, we can conclude that the factor variances have decreased over time. This could be tested separately for *Efficacy* and *Respons*.

5.1.5 Error Variances and Reliabilities

In Steps 3 and 4 of Section 4.3 I discussed functional equivalence of measurement models across time. I did not include equal error variances in this concept. Equal error variances are not necessary to compare latent variables over time. However, if the goal is to have the most parsimonious model, it may be of interest to test the hypothesis of equal error variances over time. To do so, add the lines

```
Equal Error Variances: NOSAY1 NOSAY2  
Equal Error Variances: COMPLEX1 COMPLEX2  
Equal Error Variances: NOCARE1 NOCARE2  
Equal Error Variances: TOUCH1 TOUCH2  
Equal Error Variances: INTERES1 INTERES2
```

in **ORD32.SPL** or **ORD32A.SPL**, see file **ORD34.SPL**. This gives a chi-square of 55.11 with 36 degrees of freedom. This should be compared with 38.11 with 31 degrees of freedom for the original panel model. The difference in chi-squares is 17 with 5 degrees of freedom. This difference is significant at the 1% level. So the hypothesis of equal error variances is rejected. However, this hypothesis can be tested for each item separately.

The test for NOSAY gives (see file **ORD34A.SPL**) a chi-square of 34.21 with 32 degrees of freedom. The chi-square difference is 2.45 with 1 degree of freedom. Hence, the hypothesis of equal error variances for NOSAY cannot be rejected.

The fact that the error variances of NOSAY1 and NOSAY2 are equal does not imply that their reliabilities are equal. I found in the first example that the variance of *Efficacy* decreased over time. As a consequence of this, the fitted variance of NOSAY1 and NOSAY2 will be different.

To estimate the reliabilities of NOSAY1 and NOSAY2 proceed as follows.

- From the output of **ORD34A.SPL** find the error variance of NOSAY and the specific variance of NOSAY. These are 0.92 and 0.031, respectively. Thus, the pure random measurement error variance of NOSAY is $0.92 - 0.03 = 0.89$. This is the same for NOSAY1 and NOSAY2.
- Next find the fitted variances of NOSAY1 and NOSAY2 (these are obtained if the line Print Residuals is

included in the input). These are 0.87 and 0.63, respectively.

- The reliability of NOSAY1 is now computed as $1 - 0.89/0.87 = 0.13$. For NOSAY2 I get the reliability $1 - 0.89/0.63 = 0.17$.

5.1.6 LISREL Notation

Some readers may be interested in doing the modeling using LISREL notation and syntax. Those who are not interested in this can skip this section.

Let

$$\mathbf{x} = (\text{NOSAY1}, \text{COMPLEX1}, \text{NOCARE1}, \text{TOUCH1}, \text{INTERES1}),$$

$$\mathbf{y} = (\text{NOSAY2}, \text{COMPLEX2}, \text{NOCARE2}, \text{TOUCH2}, \text{INTERES2}),$$

$$\boldsymbol{\xi} = (\text{Efficac1}, \text{Respons1}),$$

$$\boldsymbol{\eta} = (\text{Efficac2}, \text{Respons2}).$$

Then the panel model in LISREL notation is (see Jöreskog & Sörbom, 1999b, Chapter 10 or Jöreskog, et al., 2001, Chapter 1).

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & \lambda_{31} \\ 0 & \lambda_{41} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & \lambda_{31} \\ 0 & \lambda_{41} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{pmatrix}$$

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{22} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix}$$

These correspond to the three general equations in LISREL.

$$\mathbf{x} = \boldsymbol{\tau}_x + \boldsymbol{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta},$$

$$\mathbf{y} = \boldsymbol{\tau}_y + \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon},$$

$$\boldsymbol{\eta} = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}.$$

5.1.7 LISREL Syntax

An input file in LISREL syntax corresponding to **ORD32A.SPL** is the file **ORD35.LIS**

```
LISREL File for Estimating the Panel Model
SY=ORD31.DSF
SE
6 7 8 9 10 1 2 3 4 5
MO NY=5 NX=5 NE=2 NK=2 GA=FI PS=SY,FR TH=FI TX=FR TY=FR AL=FR
LK Efficac1 Respons1
LE Efficac2 Respons2
VA 1 LY(1,1) LY(5,2) LX(1,1) LX(5,2)
FR LX(2,1) LX(3,1) LX(3,2) LX(4,2)
FR GA(1,1) GA(2,2)
FR TH(1,1) TH(2,2) TH(3,3) TH(4,4) TH(5,5)
EQ LX(2,1) LY(2,1)
EQ LX(3,1) LY(3,1)
EQ LX(3,2) LY(3,2)
EQ LX(4,2) LY(4,2)
EQ TX(1) TY(1)
EQ TX(2) TY(2)
EQ TX(3) TY(3)
EQ TX(4) TY(4)
EQ TX(5) TY(5)
OU ME=WL
```

The lines

```
SE
6 7 8 9 10 1 2 3 4 5
```

are needed to order the variables such that the y-variables come first, as required by LISREL. The rest of the output is self-explanatory for users who are familiar with LISREL syntax. It can be verified that the results from **ORD35.LIS** are the same as for **ORD32A.SPL**.

6. Four-Wave Models

The previous two-wave model can be generalized to the multiwave situation when the same ordinal variables are used at more than two occasions. I do not have access to any multiwave ordinal data, so I will use generated data on four occasions. I will pretend that the variables are the six efficacy items (this time I include VOTING) measured on a four-category scale with categories having the same meaning.

6.1 A Four-Wave Model with a Single Latent Variable

The data for this example is in the file **EFFI4WS.RAW** in free format. There are $3 \times 4 = 12$ variables, where the first 3 correspond to Time 1, the next 3 to Time 2, etc.

PRELIS Step I begin by estimating the mean vector, the covariance matrix, and the asymptotic covariance matrix of the variables underlying the ordinal variables under equal thresholds for each item over time. The PRELIS command file for this is (file **EFFI4WS1.PRL**):

```

Computing ME, CM, and ACC for EFFI4WS.RAW
Under Equal Thresholds over Time
DA NI=12
LA
NOSAY1 VOTING1 COMPLEX1
NOSAY2 VOTING2 COMPLEX2
NOSAY3 VOTING3 COMPLEX3
NOSAY4 VOTING4 COMPLEX4
RA=EFFI4WS.RAW
CL NOSAY1 - COMPLEX4 1=AS 2=A 3=D 4=DS
ET NOSAY1 NOSAY2 NOSAY3 NOSAY4
ET VOTING1 VOTING2 VOTING3 VOTING4
ET COMPLEX1 COMPLEX2 COMPLEX3 COMPLEX4
OU MA=CM ME=EFFI4WS.ME CM=EFFI4WS.CM AC=EFFI4WS.ACC

```

The output reveals that there are 2357 cases and that the univariate marginal parameters are

```
Total Sample Size(N) = 2357
```

```
Univariate Marginal Parameters
```

Variable	Mean	St. Dev.	Thresholds		
-----	----	-----	-----	-----	-----
NOSAY1	-0.032	1.090	0.000	1.000	2.058
VOTING1	-0.003	0.935	0.000	1.000	1.850
COMPLEX1	-0.008	1.001	0.000	1.000	1.946
NOSAY2	0.818	1.192	0.000	1.000	2.058
VOTING2	0.582	1.019	0.000	1.000	1.850
COMPLEX2	0.599	1.097	0.000	1.000	1.946
NOSAY3	2.035	1.399	0.000	1.000	2.058
VOTING3	1.402	1.134	0.000	1.000	1.850
COMPLEX3	1.455	1.204	0.000	1.000	1.946
NOSAY4	3.800	1.666	0.000	1.000	2.058
VOTING4	2.470	1.199	0.000	1.000	1.850
COMPLEX4	2.736	1.430	0.000	1.000	1.946

Note that there is an increasing trend in the means of the underlying variables over time.

LISREL Step The measurement model assumes that NOSAY, VOTING, and COMPLEX are indicators of a single latent variable *Efficacy*. This measurement model is employed at each time point and is assumed to be invariant over time. Both the intercepts and the factor loadings are assumed to be the same across time. In addition, it is assumed that VOTING and COMPLEX, but not NOSAY, contain specific factors denoted *Voting* and *Complex*. These specific factors are constant over time and uncorrelated with *Efficacy* at all times.

First I focus on the estimation of the means and variances of *Efficacy* over time without assuming any structural model. A SIMPLIS command file to estimate such a model is (file **EFFI4WS2.SPL**):

```

SIMPLIS Input for Four-Wave Model
One Factor
Observed Variables:
NOSAY1 VOTING1 COMPLEX1
NOSAY2 VOTING2 COMPLEX2
NOSAY3 VOTING3 COMPLEX3

```

NOSAY4 VOTING4 COMPLEX4
Means from File EFFI4WS.ME
Covariance Matrix from File EFFI4WS.CM
Asymptotic Covariance Matrix from File EFFI4WS.ACC
Sample Size: 2357
Latent Variables: Efficac1 Efficac2 Efficac3 Efficac4
Voting Complex

Relationships

NOSAY1 - COMPLEX1 = CONST (1)*Efficac1
NOSAY1 = 1*Efficac1

NOSAY2 - COMPLEX2 = CONST (1)*Efficac2
NOSAY2 = 1*Efficac2

NOSAY3 - COMPLEX3 = CONST (1)*Efficac3
NOSAY3 = 1*Efficac3

NOSAY4 - COMPLEX4 = CONST (1)*Efficac4
NOSAY4 = 1*Efficac4

Set CONST -> NOSAY1 = CONST -> NOSAY2
Set CONST -> VOTING1 = CONST -> VOTING2
Set CONST -> COMPLEX1 = CONST -> COMPLEX2

Set CONST -> NOSAY1 = CONST -> NOSAY3
Set CONST -> VOTING1 = CONST -> VOTING3
Set CONST -> COMPLEX1 = CONST -> COMPLEX3

Set CONST -> NOSAY1 = CONST -> NOSAY4
Set CONST -> VOTING1 = CONST -> VOTING4
Set CONST -> COMPLEX1 = CONST -> COMPLEX4

Set Efficac1 -> VOTING1 = Efficac2 -> VOTING2
Set Efficac1 -> COMPLEX1 = Efficac2 -> COMPLEX2

Set Efficac1 -> VOTING1 = Efficac3 -> VOTING3
Set Efficac1 -> COMPLEX1 = Efficac3 -> COMPLEX3

Set Efficac1 -> VOTING1 = Efficac4 -> VOTING4
Set Efficac1 -> COMPLEX1 = Efficac4 -> COMPLEX4

VOTING1 VOTING2 VOTING3 VOTING4 = 1*Voting
COMPLEX1 COMPLEX2 COMPLEX3 COMPLEX4 = 1*Complex

Set the covariances of Voting and Complex to 0

Efficac1 = 0*CONST
Efficac2 = CONST
Efficac3 = CONST
Efficac4 = CONST

Set the covariances of Efficac1 - Efficac4 free
Set the covariances between Voting - Complex and Efficac1 - Efficac4 to 0
Method of Estimation: Weighted Least Squares
End of Problem

A difference between this example and the two-wave models considered previously is that the specific factors are now modeled directly as factors affecting the underlying variables rather than as error covariances, see the lines marked A and B. It is possible to specify them as error covariances also, but this is tedious in the SIMPLIS command language because one has to define six error covariances for each of VOTING and COMPLEX and one has to specify the equality of these six error covariances as well.

Without (1)* in front of *Efficacy* in the measurement model, the LISREL iterations do not converge. But this is only a starting value problem. All one needs to do is to give LISREL some help to get iterations started. This is done by specifying 1 as a starting value for the factor loading of *Efficacy*. Note the difference between (1)* and 1* as in lines A and B. The latter is a fixed value.

I leave it to the reader to contemplate most of the information in the output. Here I will confine myself to giving only the covariance matrix and the mean vector of the latent variables.

Covariance Matrix of Independent Variables

	Effiac1	Effiac2	Effiac3	Effiac4	Voting	Complex
	-----	-----	-----	-----	-----	-----
Effiac1	0.785 (0.021) 37.143					
Effiac2	0.701 (0.019) 36.303	1.078 (0.022) 49.499				
Effiac3	0.747 (0.024) 30.800	1.135 (0.021) 54.591	1.612 (0.025) 63.237			
Effiac4	0.806 (0.034) 23.895	1.267 (0.031) 41.006	1.743 (0.031) 56.230	2.347 (0.044) 53.693		
Voting	- -	- -	- -	- -	0.213 (0.011) 19.556	
Complex	- -	- -	- -	- -	- -	0.301 (0.014) 21.187

Mean Vector of Independent Variables

	Effiac1	Effiac2	Effiac3	Effiac4	Voting	Complex
	-----	-----	-----	-----	-----	-----
	- -	0.831 (0.026) 32.272	2.040 (0.025) 81.475	3.778 (0.028) 134.806	- -	- -

which shows that there is an increasing trend in both means and variances of the latent variables over time. It also shows that there are specific factors in VOTING and COMPLEX.

A Simplex Model Next I assume that *Efficacy* is generated by an autoregressive process:

$$Efficacy_t = \alpha_t^{(E)} + \beta_t^{(E)} Efficacy_{t-1}, \quad t = 2, 3, 4$$

This kind of model is sometimes called a Simplex model, see, e.g., Jöreskog & Sörbom, 1999b, pp. 230-238.

A SIMPLIS command file to estimate this model is (file **EFFI4WS3.SPL**):

```
SIMPLIS Input for Four-Wave Model
Single Simplex
...
...
...
Set the covariance between Voting and Complex to 0

Efficac2 = CONST Efficac1
Efficac3 = CONST Efficac2
Efficac4 = CONST Efficac3

No x-variables
Admissibility Check Off
End of Problem
```

The lines down to and including

```
Set the covariance between Voting and Complex to 0
```

are the same as before.

This kind of model is best handled as a model with only y - and η -variables in LISREL (LISREL submodel 3B, see Jöreskog & Sörbom, 1999c, Chapter 6). The SIMPLIS command to specify this is

```
No x-variables
```

The line

```
Admissibility Check Off
```

is needed to make the iterations converge (LISREL checks the admissibility of parameters after 50 iterations and stops if this check is not set off, see Jöreskog & Sörbom, 1999c, pp. 322-323).

The chi-square for this model is 182.52 with 61 degrees of freedom, compared with the previous 177.56 with 58 degrees of freedom. The difference 4.96 with 3 degrees of freedom means that the Simplex model cannot be rejected.

6.2 Four-Wave Model with Two Latent Variables

The previous four-wave model can be generalized to a model with six observed and two latent variables. This generalization is straightforward but the input files are very long and elaborate. Rather than listing these files here, I leave it for the interested reader to carry out the analysis.

The data is in the file **EFFI4W.RAW** in free format. There are $6 \times 4 = 24$ variables, where the first 6 correspond to Time 1, the next 6 to Time 2, etc.

PRELIS Step I begin by estimating the mean vector, the covariance matrix, and the asymptotic covariance matrix of the variables underlying the ordinal variables under equal thresholds for each item over time. The PRELIS command file for this is **EFFI4W1.PRL**.

LISREL Step The measurement model is shown in Fig. 5. This measurement model is employed

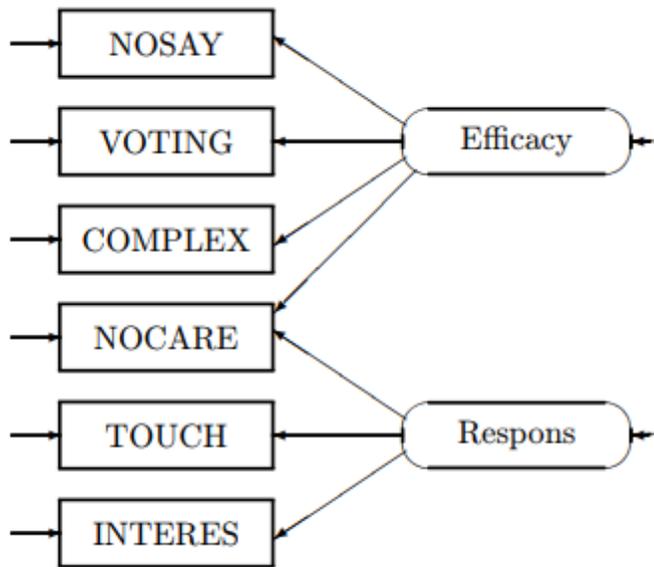


Figure 5: Six Variables Measurement Model for Efficacy and Respons

at each time point and is assumed to be invariant over time. In addition, I assume that VOTING and COMPLEX contain specific factors denoted Voting and Complex. These specific factors are constant over time and uncorrelated with the factors *Efficacy* and *Respons* at all times. *Efficacy* and *Respons* are themselves contemporaneously correlated.

The structural model assumes that *Efficacy* and *Respons* are generated by an autoregressive process:

$$Efficacy_t = \alpha_t^{(E)} + \beta_t^{(E)} Efficacy_{t-1}, \quad t = 2, 3, 4$$

$$Respons_t = \alpha_t^{(R)} + \beta_t^{(R)} Respons_{t-1}, \quad t = 2, 3, 4$$

A SIMPLIS command file to estimate this model is **EFFI4W2.SPL**. This command file is quite large and elaborate. For this kind of models, the LISREL command language is more convenient. Users who have mastered the LISREL command language can use the shorter input given in **EFFI4W3.LIS**.