Scientific Software International

Chi-square statistics produced by LISREL

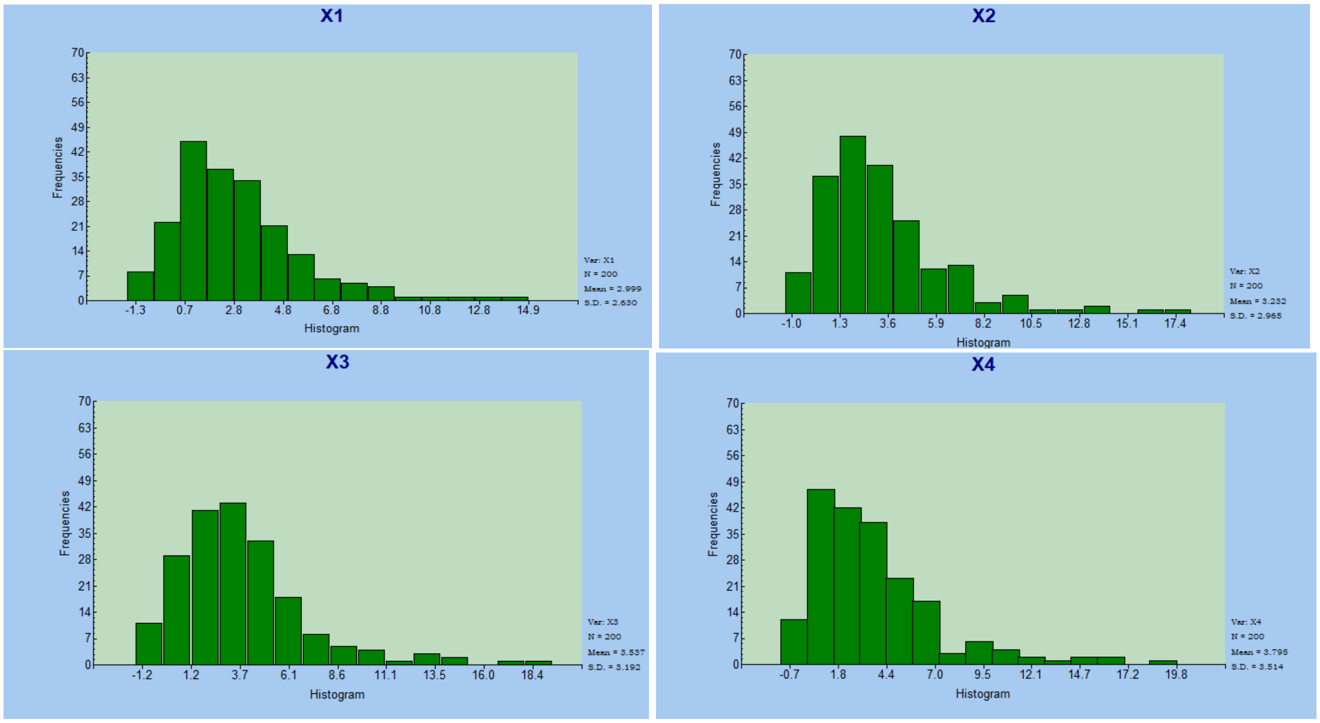
SEM analyses typically assume that the observed variables have a multivariate normal distribution. This assumption is questionable in many cases. Although the maximum likelihood parameter estimates are considered to be robust against non-normality, their standard errors and chi-squares are affected by non-normality.

In the case of non-normality, LISREL calculates several different methods to obtain more correct standard errors and chi-square statistics. For illustration purposes, we use the dataset **chi2test.lsf** consisting of simulated data for four variables.

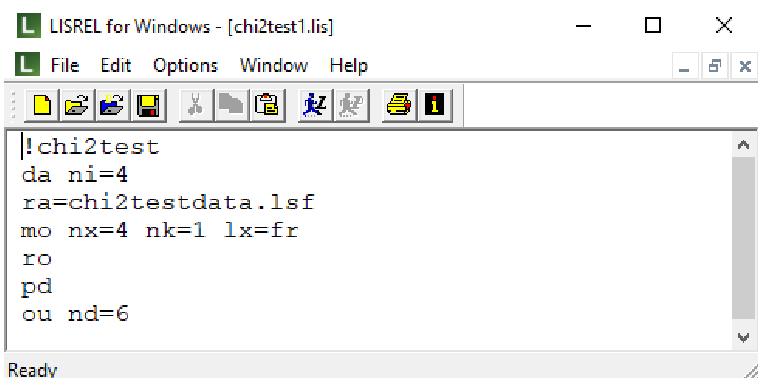
	X1	X2	X3	X4
1	0.79	1.48	1.43	3.10
2	1.68	3.16	2.72	-1.23
3	11.01	12.39	14.02	16.84
4	4.27	4.85	3.21	3.24
5	4.36	2.05	2.61	4.35
6	1.58	1.48	-0.61	1.61
7	7.65	9.15	7.88	9.55
8	5.94	5.77	4.87	5.54
9	6.57	8.80	8.41	9.40
10	1.60	1.34	2.88	1.30
11	6.33	5.63	7.59	7.24
12	3.20	5.03	3.98	5.40
13	0.30	-1.18	1.04	1.83
14	2.80	4.89	4.18	3.68
15	1.83	1.19	4.20	3.78
16	3.26	0.84	2.60	1.96
17	2.19	1.18	-0.27	-0.02
18	2.19	3.00	3.23	4.37
19	2.35	2.56	2.46	1.38
20	0.04	-0.07	-0.17	0.62

Ready

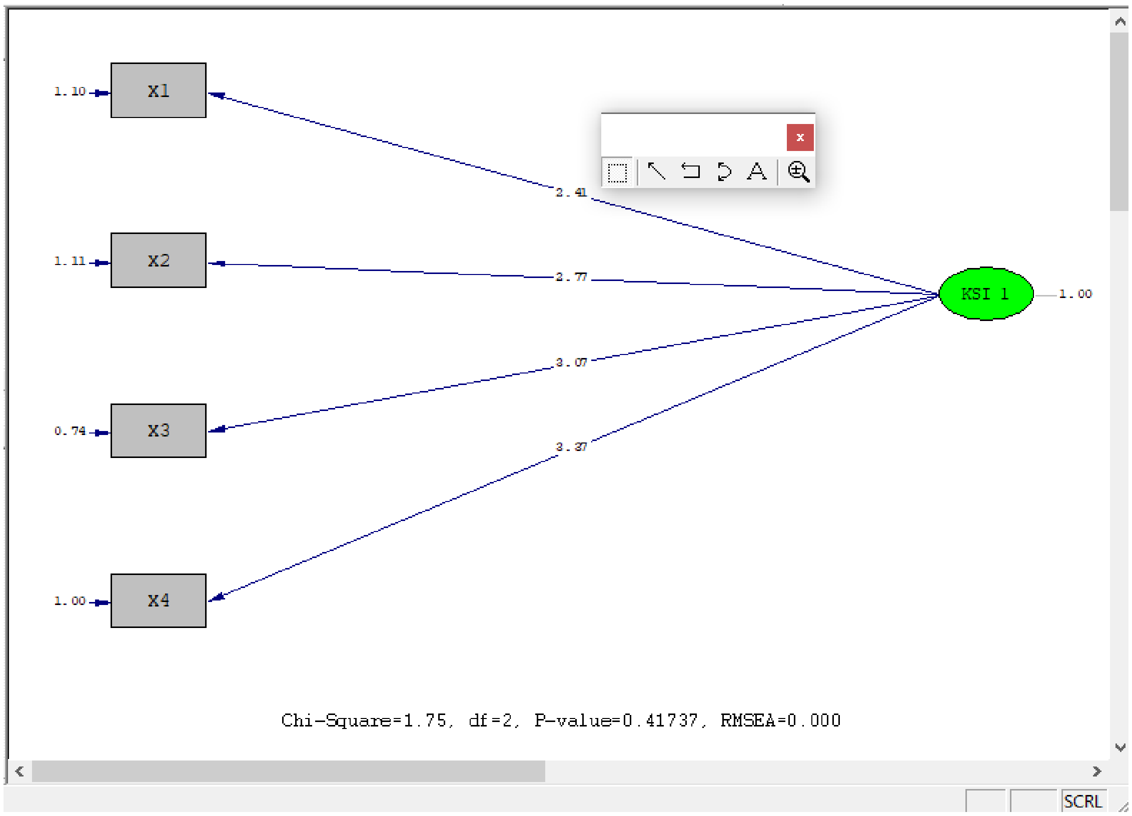
Univariate graphs of the four variables are shown below. Clearly, none of them have a normal distribution: all four are skewed with a long tail to the right.



We assume that the four X-variables measure the same latent construct, as expressed in the syntax file (**chi2test.lis**) shown below.



The following path diagram is obtained for this model.



From the output file, we obtain the following descriptive statistics. The four variables are reasonably similar in terms of skewness, z-scores, chi-squares and *p* -values. We note that the hypothesis of zero skewness and kurtosis is rejected for all four variables. It is therefore recommended to use the maximum likelihood method with robust standard errors and chi-squares, which is called Robust Maximum Likelihood (note the inclusion of the keyword 'ro' in the syntax file).

Total Sample Size(N) = 200

Univariate Summary Statistics for Continuous Variables

Variable	Mean	St. Dev.	Skewness	Kurtosis	Minimum	Freq.	Maximum	Freq.
X1	2.999	2.630	1.369	2.711	-1.624	1	14.118	1
X2	3.232	2.965	1.580	3.541	-1.371	1	16.458	1
X3	3.537	3.192	1.580	3.544	-1.611	1	17.406	1
X4	3.795	3.514	1.658	3.533	-1.232	1	18.847	1

Test of Univariate Normality for Continuous Variables

	Skewness		Kurtosis		Skewness and Kurtosis	
Variable	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
X1	6.364	0.000	4.099	0.000	57.310	0.000
X2	7.000	0.000	4.658	0.000	70.695	0.000
X3	6.999	0.000	4.660	0.000	70.705	0.000
X4	7.218	0.000	4.654	0.000	73.759	0.000

Means

X1	X2	X3	X4
2.999	3.232	3.537	3.795

Standard Deviations

X1	X2	X3	X4
2.630	2.965	3.192	3.514

LISREL Estimates (Robust Maximum Likelihood)

LAMBDA-X

	KSI 1
X1	2.410570
	(0.216150)
	11.152316
X2	2.772531
	(0.261374)
	10.607521
X3	3.072910
	(0.272318)
	11.284279
X4	3.368778
	(0.300370)
	11.215445

PHI

KSI 1
1.000000

THETA-DELTA

X1	X2	X3	X4
1.104981	1.106729	0.744808	0.996212
(0.127365)	(0.141202)	(0.113143)	(0.149538)
8.675706	7.837926	6.582890	6.661920

Squared Multiple Correlations for X - Variables

X1	X2	X3	X4
0.840224	0.874145	0.926891	0.919302

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Log-likelihood Values

                        Estimated Model          Saturated Model
                        ---------------          ---------------
 Number of free parameters(t)         8                       10
 -2ln(L)                       1504.214                 1502.215
 AIC (Akaike, 1974)*           1520.214                 1522.215
 BIC (Schwarz, 1978)*          1546.560                 1555.148

*LISREL uses AIC= 2t - 2ln(L) and BIC = tln(N)- 2ln(L)


                           Goodness-of-Fit Statistics

 Degrees of Freedom for (C1)-(C3),(C5)                  2
 Maximum Likelihood Ratio Chi-Square (C1)               1.999170 (P = 0.3680)
 Browne's (1984) ADF Chi-Square (C2_NT)                 2.066047 (P = 0.3559)
 Browne's (1984) ADF Chi-Square (C2_NNT)                1.627474 (P = 0.4432)
 Satorra-Bentler (1988) Scaled Chi-Square (C3)          1.747588 (P = 0.4174)
 Satorra-Bentler (1988) Adjusted Chi-Square             1.722566 (P = 0.4160)
 Degrees of Freedom for C4                               1.971
 Chi-Square Scaled and Shifted (C5)                     1.749401 (P = 0.4170)
 P-Value of C1 under Non-Normality                                = 0.4159

 Estimated Non-centrality Parameter (NCP)               0.0
 90 Percent Confidence Interval for NCP                 (0.0 ; 7.242472)

 Minimum Fit Function Value                             0.0100461
 Population Discrepancy Function Value (F0)             0.0
 90 Percent Confidence Interval for F0                  (0.0 ; 0.0363943)
 Root Mean Square Error of Approximation (RMSEA)        0.0
 90 Percent Confidence Interval for RMSEA               (0.0 ; 0.134897)
 P-Value for Test of Close Fit (RMSEA < 0.05)           0.575235

 Expected Cross-Validation Index (ECVI)                 0.0904523
 90 Percent Confidence Interval for ECVI                (0.0904523 ; 0.126847)
 ECVI for Saturated Model                               0.100503
 ECVI for Independence Model                            5.471382

 Chi-Square for Independence Model (6 df)             1080.805

 Normed Fit Index (NFI)                                 0.998383
 Non-Normed Fit Index (NNFI)                            1.000705
 Parsimony Normed Fit Index (PNFI)                      0.332794
 Comparative Fit Index (CFI)                            1.000000
 Incremental Fit Index (IFI)                            1.000234
 Relative Fit Index (RFI)                               0.995149

 Critical N (CN)                                     1049.887

 Root Mean Square Residual (RMR)                        0.0322179
 Standardized RMR                                       0.00362018
 Goodness of Fit Index (GFI)                            0.994836
 Adjusted Goodness of Fit Index (AGFI)                  0.974178
 Parsimony Goodness of Fit Index (PGFI)                 0.198967
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If the observed variables are non-normal, one can use the same formula from Browne (1984) using an asymptotic covariance matrix (ACM) estimated under non-normality. This chi-square, often called the ADF (Asymptotically Distribution Free) chi-square statistic, is denoted C2_NNT in LISREL[i] 11. It has been found in simulation studies that the ADF statistic does not work well because it is difficult to estimate the ACM [ii] accurately unless *N* is huge, see *e.g*, Curran, West, & Finch (1996).

Satorra & Bentler (1988) proposed another approximate chi-square statistic C3, which is C1 multiplied by a scale factor which is estimated from the sample and involves estimates of the ACM both under normality and non-normality. The scale factor is estimated such that C3 has an asymptotically correct mean even though it does not have an asymptotic chi-square distribution. In practice, C3 is conceived of as a way of correcting C1 for the effects of non-normality and C3 is often used as it performs better than the ADF test C2_NT in LISREL, particularly if *N* is not very large, see *e.g.*, Hu, Bentler, & Kano (1992).

Satorra & Bentler (1988) also mentioned the possibility of using a Satterthwaite (1941) type correction which adjusts C1 such that the corrected value has the correct asymptotic mean and variance. This type of fit measure has not been much investigated, neither for continuous nor for ordinal variables. However, this type of chi-square fit statistic has been implemented in LISREL, where it is denoted C4. C1 and C2_NT are the same as before, with robust estimation LISREL also gives C2_NNT, C3 and C4 so that one can see what the effect of non-normality is. In particular, the difference C2_NNT - C2_NT can be viewed as an effect of non-normality.

For the current data, these are given as

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Browne's (1984) ADF Chi-Square (C2_NT)                 2.066047 (P = 0.3559)
Browne's (1984) ADF Chi-Square (C2_NNT)                1.627474 (P = 0.4432)
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Note that C4 has its own degrees of freedom which is different from the model degrees of freedom. The degrees of freedom for C4 is given as a fractional number and uses this fractional degrees of freedom to compute the *P*-value for C4.

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[i] The ACM is an estimate of the covariance matrix of the sample variances and covariances. Under non-normality this involves estimates of fourth-order moments.

[ii] In previous versions of LISREL, C_2(NT) and C_2(NNT) was called C2 and C4, respectively.