Confirmatory factor analysis of ordinal data

Contents

1. Assumption of normality

While all the observed variables are continuous, maximum likelihood estimation has an underlying assumption of multivariate normality. With non-normal continuous data, ML produces relatively accurate parameter estimates, but the bias in chi-square and standard errors increases with non-normality.

While WLS estimation produces accurate parameter estimates, there is a tendency to underestimate standard errors and overestimate goodness-of-fit measures. Larger or more complex models, or greater nonnormality, also sometimes causes a failure of WLS to converge.

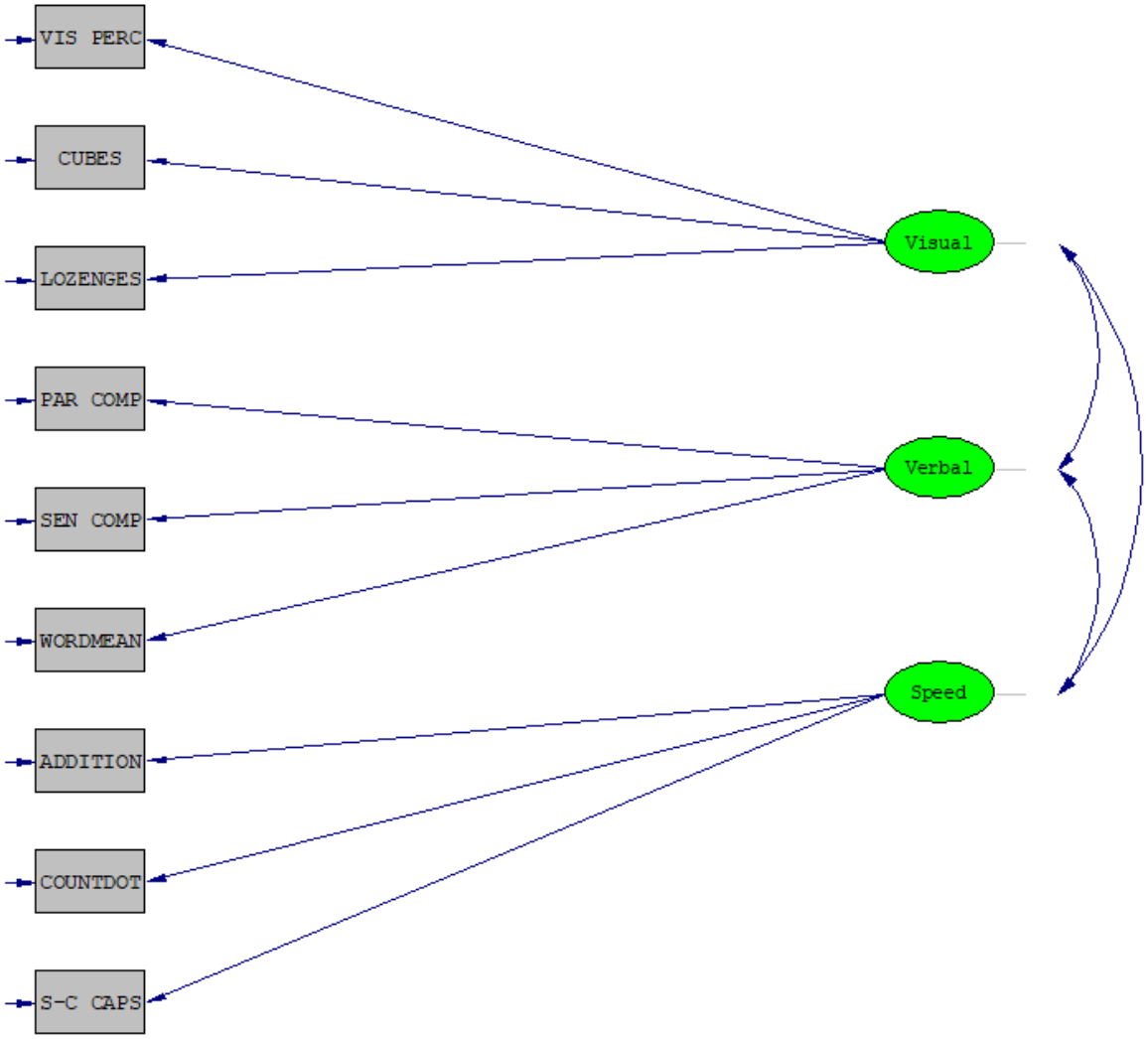
Modifying the full WLS approach to utilize only the diagonal elements of the asymptotic covariance matrix in an approach that became known as diagonally weighted least squares (DWLS). DWLS produces parameter estimates that appear to be less distorted by nonnormality than are ML estimates. Further modifications include using adjustment formulas to produce more accurate results, correcting standard errors and χ^2-values for bias due to nonnormality. This approach is commonly referred to as robust WLS or robust DWLS.

In this example (Holzinger & Swineford (1939)) nine variables were selected to measure three latent factors: Space, Verbal and Visual. The group of interest in this example consists of 145 eighth-grade children from the Grant-White school in Chicago.

To take a closer look at the impact of the non-normality of these variables on the estimates, standard error estimates and the values of the goodness-of-fit statistics, we now fit a confirmatory factor analysis model to these data by using three estimation methods:

- ML estimation with robust standard errors based on a covariance matrix
- ML estimation using the asymptotic covariance matrix (robust ML), and
- WLS estimation using the asymptotic covariance matrix

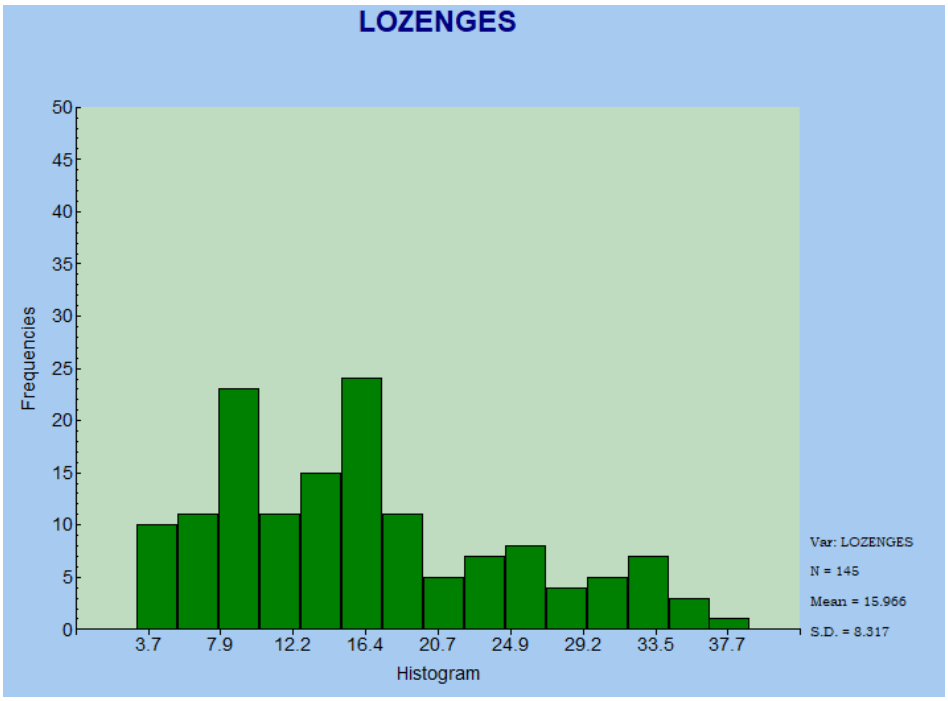
A conceptual path diagram of the model to be fitted is shown below.

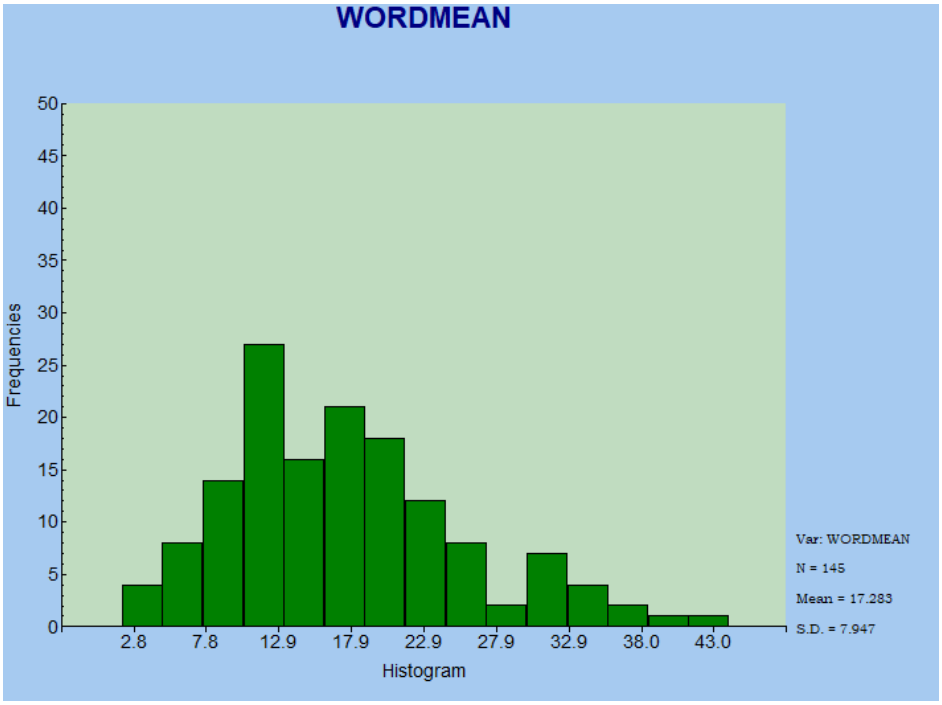


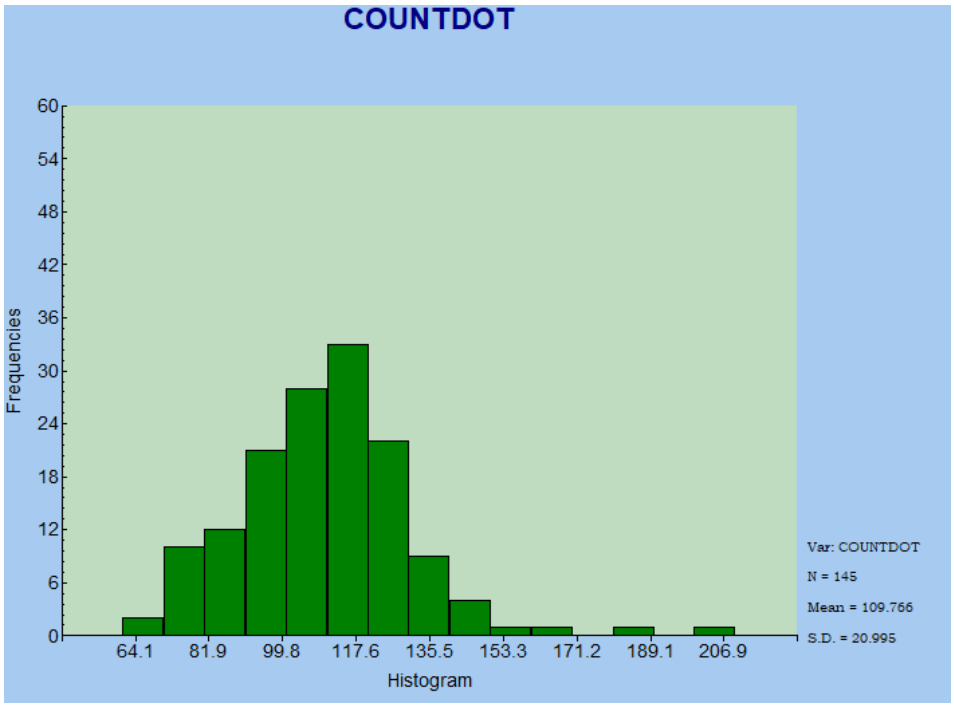
2. Data exploration

As a first step, we take a closer look at the distributions of the observed variables we intend to use. The data are contained in the LSF file **NPV.LSF**.

Bar charts for the variables LOZENGES, WORDMEAN and COUNTDOT are shown below. Clearly, these three variables are not normally distributed.


LOZENGES
Frequencies
50
45
40
35
30
25
20
15
10
5
0
3.7
7.9
12.2
16.4
20.7
24.9
29.2
33.5
37.7
Histogram
Var: LOZENGES
N = 145
Mean = 15.966
S.D. = 8.317


WORDMEAN
Frequencies
50
45
40
35
30
25
20
15
10
5
0
2.8
7.8
12.9
17.9
22.9
27.9
32.9
38.0
43.0
Histogram
Var: WORDMEAN
N = 145
Mean = 17.283
S.D. = 7.947



A data screening confirms these suspicions:

Total Sample Size(N) = 145

Univariate Summary Statistics for Continuous Variables

Variable	Mean	St. Dev.	Skewness	Kurtosis	Minimum	Freq.	Maximum	Freq.
VISPERC	29.579	6.914	-0.119	-0.046	11.000	1	51.000	1
CUBES	24.800	4.445	0.239	0.872	9.000	1	37.000	2
LOZENGES	15.966	8.317	0.623	-0.454	3.000	2	36.000	1
PARCOMP	9.952	3.375	0.405	0.252	1.000	1	19.000	1
SENCOMP	18.848	4.649	-0.550	0.221	4.000	1	28.000	1
WORDMEAN	17.283	7.947	0.729	0.233	2.000	1	41.000	1
ADDITION	90.179	23.782	0.163	-0.356	30.000	1	149.000	1
COUNTDOT	109.766	20.995	0.698	2.283	61.000	1	200.000	1
SCCAPS	191.779	37.035	0.200	0.515	112.000	1	333.000	1

Test of Univariate Normality for Continuous Variables

	Skewness		Kurtosis		Skewness and Kurtosis	
Variable	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
VISPERC	-0.604	0.546	0.045	0.964	0.367	0.833
CUBES	1.202	0.229	1.843	0.065	4.842	0.089
LOZENGES	2.958	**0.003**	-1.320	0.187	10.491	**0.005**
PARCOMP	1.995	**0.046**	0.761	0.447	4.559	0.102
SENCOMP	-2.646	**0.008**	0.693	0.489	7.483	**0.024**
WORDMEAN	3.385	**0.001**	0.720	0.472	11.977	**0.003**
ADDITION	0.826	0.409	-0.937	0.349	1.560	0.458
COUNTDOT	3.263	**0.001**	3.325	0.001	21.699	**0.000**
SCCAPS	1.008	0.313	1.273	0.203	2.638	0.267

```
 Relative Multivariate Kurtosis = 1.072

 Test of Multivariate Normality for Continuous Variables

             Skewness                    Kurtosis           Skewness and Kurtosis

      Value  Z-Score P-Value     Value  Z-Score P-Value      Chi-Square P-Value
     ------  ------- -------   -------  ------- -------      ---------- -------
     11.733    5.426   0.000   106.098    3.023   0.003          38.579   0.000
```

Based on these results, we anticipate that relatively accurate parameter estimates may be obtained using maximum likelihood estimation, but that increased bias in chi-square and standard errors may occur due to the non-normality in some of the observed variables.

3. Maximum likelihood estimation with robust standard errors

SIMPLIS syntax for the first method is given in **hoswgw1.spl**, which can be found in the **SIMPLIS Examples\Factor Analysis and PCA** folder. The analysis is based on the covariance matrix saved in the file **HOSWGW.CM**. The SIMPLIS syntax below specifies the fitting of a confirmatory factor analysis model with three latent variables.

```
Nine Psychological Variables - A Confirmatory Factor Analysis
Estimating Model by ML with Robust Standard Errors
Observed Variables
   'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
    ADDITION COUNTDOT 'S-C CAPS'
Covariance Matrix From File HOSWGW.CM
Sample Size 145
Latent Variables: Visual Verbal Speed
Relationships:
   'VIS PERC' - LOZENGES = Visual
   'PAR COMP' - WORDMEAN = Verbal
   ADDITION - 'S-C CAPS' = Speed
Number of Decimals = 3
Print Residuals
Path Diagram
End of Problem
```

Partial results for the analysis are shown below.

```
LISREL Estimates (Maximum Likelihood)
         Measurement Equations

  VIS PERC = 4.678*Visual, Errorvar.= 25.915, R² = 0.458
 Standerr  (0.624)                  (4.582)
 Z-values   7.499                   5.656
 P-values   0.000                   0.000

    CUBES = 2.296*Visual, Errorvar.= 14.486, R² = 0.267
 Standerr  (0.408)                  (1.981)
 Z-values   5.622                   7.313
 P-values   0.000                   0.000

 LOZENGES = 5.769*Visual, Errorvar.= 35.896, R² = 0.481
 Standerr  (0.751)                  (6.660)
 Z-values   7.684                   5.390
 P-values   0.000                   0.000
```

```
 PAR COMP = 2.922*Verbal, Errorvar.= 2.857 , R² = 0.749
 Standerr  (0.237)                  (0.589)
 Z-values   12.312                   4.853
 P-values   0.000                    0.000

 SEN COMP = 3.856*Verbal, Errorvar.= 6.750 , R² = 0.688
 Standerr  (0.333)                  (1.165)
 Z-values   11.589                   5.793
 P-values   0.000                    0.000

 WORDMEAN = 6.567*Verbal, Errorvar.= 20.032, R² = 0.683
 Standerr  (0.569)                  (3.419)
 Z-values   11.532                   5.859
 P-values   0.000                    0.000

 ADDITION = 15.676*Speed, Errorvar.= 319.855, R² = 0.434
 Standerr  (2.012)                  (48.753)
 Z-values   7.792                    6.561
 P-values   0.000                    0.000

 COUNTDOT = 16.710*Speed, Errorvar.= 161.580, R² = 0.633
 Standerr  (1.752)                  (38.166)
 Z-values   9.535                    4.234
 P-values   0.000                    0.000

 S-C CAPS = 25.956*Speed, Errorvar.= 697.907 , R² = 0.491
 Standerr  (3.117)                  (116.521)
 Z-values   8.328                    5.990
 P-values   0.000                    0.000
```

Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	24
Maximum Likelihood Ratio Chi-Square (C1)	51.190 (P = 0.00100)
Browne's (1984) ADF Chi-Square (C2_NT)	48.617 (P = 0.00212)
Estimated Non-centrality Parameter (NCP)	27.190
90 Percent Confidence Interval for NCP	(10.357 ; 51.765)
Minimum Fit Function Value	0.355
Population Discrepancy Function Value (F0)	0.189
90 Percent Confidence Interval for F0	(0.0719 ; 0.359)
Root Mean Square Error of Approximation (RMSEA)	0.0887
90 Percent Confidence Interval for RMSEA	(0.0547 ; 0.122)
P-Value for Test of Close Fit (RMSEA < 0.05)	0.0327

Summary Statistics for Standardized Residuals

Smallest Standardized Residual =	-3.020
Median Standardized Residual =	0.000
Largest Standardized Residual =	4.965

```
Stemleaf Plot

 - 3|00
 - 2|9311
 - 1|7322
 - 0|876552110000000000
   0|111336689
   1|248
   2|139
   3|
   4|5
   5|0
 Largest Negative Standardized Residuals
 Residual for         ADDITION and         LOZENGES   -2.973
 Residual for         COUNTDOT and         PAR COMP   -3.020
 Residual for         S-C CAPS and         COUNTDOT   -2.904
 Largest Positive Standardized Residuals
 Residual for         COUNTDOT and         ADDITION    4.965
 Residual for         S-C CAPS and         VIS PERC    4.495
 Residual for         S-C CAPS and         SEN COMP    2.904
```

4. Maximum likelihood estimation with robust standard errors using the asymptotic covariance matrix

SIMPLIS syntax for the second method, using maximum likelihood estimation with robust standard errors, is shown below (**hoswgw2.spl**). The asymptotic covariance matrix is read in from the file **HOSWGW.ACC**. Maximum likelihood estimation is specified on the Method command line.

```
Estimating Model by ML with Robust Standard Errors
Observed Variables
   'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
    ADDITION COUNTDOT 'S-C CAPS'
Covariance Matrix From File HOSWGW.CM
Asymptotic Covariance Matrix From File HOSWGW.ACC
Sample Size 145
Latent Variables: Visual Verbal Speed
Relationships:
   'VIS PERC' - LOZENGES = Visual
   'PAR COMP' - WORDMEAN = Verbal
   ADDITION - 'S-C CAPS' = Speed
Number of Decimals = 3
Print Residuals
Method: Maximum Likelihood
Path Diagram
End of Problem
```

Inspection of the results shows the same measurement equation estimates, but also a slight reduction in the RMSEA point estimate, an increase in the P-value for the test of close fit, and a reduction in the size of the largest standardized residuals when compared to the previous analysis.

```
LISREL Estimates (Robust Maximum Likelihood)
         Measurement Equations

  VIS PERC = 4.678*Visual, Errorvar.= 25.915, R² = 0.458
 Standerr  (0.696)                  (6.048)
 Z-values   6.726                   4.285
 P-values   0.000                   0.000
```

```
   CUBES = 2.296*Visual, Errorvar.= 14.486, R² = 0.267
Standerr  (0.377)                  (2.241)
Z-values   6.096                    6.465
P-values   0.000                    0.000

LOZENGES = 5.769*Visual, Errorvar.= 35.896, R² = 0.481
Standerr  (0.728)                  (6.878)
Z-values   7.925                    5.219
P-values   0.000                    0.000

PAR COMP = 2.922*Verbal, Errorvar.= 2.857 , R² = 0.749
Standerr  (0.251)                  (0.596)
Z-values   11.639                   4.797
P-values   0.000                    0.000

SEN COMP = 3.856*Verbal, Errorvar.= 6.750 , R² = 0.688
Standerr  (0.332)                  (1.146)
Z-values   11.617                   5.888
P-values   0.000                    0.000

WORDMEAN = 6.567*Verbal, Errorvar.= 20.032, R² = 0.683
Standerr  (0.575)                  (3.726)
Z-values   11.426                   5.376
P-values   0.000                    0.000

ADDITION = 15.676*Speed, Errorvar.= 319.855, R² = 0.434
Standerr  (1.836)                  (42.258)
Z-values   8.536                    7.569
P-values   0.000                    0.000

COUNTDOT = 16.710*Speed, Errorvar.= 161.580, R² = 0.633
Standerr  (1.781)                  (44.819)
Z-values   9.381                    3.605
P-values   0.000                    0.000

S-C CAPS = 25.956*Speed, Errorvar.= 697.907 , R² = 0.491
Standerr  (3.088)                  (109.637)
Z-values   8.406                    6.366
P-values   0.000                    0.000
```

Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C3),(C5)	24
Maximum Likelihood Ratio Chi-Square (C1)	51.190 (P = 0.00100)
Browne's (1984) ADF Chi-Square (C2_NT)	48.617 (P = 0.00212)
Browne's (1984) ADF Chi-Square (C2_NNT)	64.207 (P = 0.00002)
Satorra-Bentler (1988) Scaled Chi-Square (C3)	49.717 (P = 0.00154)
Satorra-Bentler (1988) Adjusted Chi-Square (C4)	34.893 (P = 0.00600)
Degrees of Freedom for C4	16.844
Chi-Square Scaled and Shifted (C5)	45.545 (P = 0.00502)
P-Value of C1 under Non-Normality	= 0.0077
Estimated Non-centrality Parameter (NCP)	25.717
90 Percent Confidence Interval for NCP	(9.280 ; 49.909)
Minimum Fit Function Value	0.355
Population Discrepancy Function Value (F0)	0.179
90 Percent Confidence Interval for F0	(0.0644 ; 0.347)
Root Mean Square Error of Approximation (RMSEA)	0.0863
90 Percent Confidence Interval for RMSEA	(0.0518 ; 0.120)

```
 P-Value for Test of Close Fit (RMSEA < 0.05)          0.0428

Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -3.292
   Median Standardized Residual =    0.000
  Largest Standardized Residual =    3.957

 Stemleaf Plot

 - 3|31
 - 2|1000
 - 1|642
 - 0|97753210000000000000
   0|1123447889
   1|47
   2|35
   3|1
   4|0
 Largest Negative Standardized Residuals
 Residual for         ADDITION and         LOZENGES   -3.053
 Residual for         S-C CAPS and         COUNTDOT   -3.292
 Largest Positive Standardized Residuals
 Residual for         COUNTDOT and         ADDITION    3.054
 Residual for         S-C CAPS and         VIS PERC    3.957
```

5. Weighted Least Squares estimation

The final analysis utilizes weighted least squares estimation. The only change in this syntax file (**hoswgw3.spl**), compared to the previous example, is in the Method command line where weighted least squares is specified.

```
Nine Psychological Variables - A Confirmatory Factor Analysis
Estimating Model by WLS
Observed Variables
   'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
    ADDITION COUNTDOT 'S-C CAPS'
Covariance Matrix From File HOSWGW.CM
Asymptotic Covariance Matrix From File HOSWGW.ACC
Sample Size 145
Latent Variables: Visual Verbal Speed
Relationships:
   'VIS PERC' - LOZENGES = Visual
   'PAR COMP' - WORDMEAN = Verbal
    ADDITION - 'S-C CAPS' = Speed
Number of Decimals = 3
Print Residuals
Path Diagram
Method of Estimation: Weighted Least Squares
End of Problem
```

Results for this analysis differ from the previous two analyses, both in estimates and *t*-values. The estimates of the factor loadings associated with the latent variable Visual and their associated standard errors are larger than before, while those for the other two variables have also changed, with some estimates smaller or larger than before.

```
LISREL Estimates (Weighted Least Squares)
         Measurement Equations


 VIS PERC = 4.406*Visual, Errorvar.= 18.464, R² = 0.513
 Standerr  (0.563)                  (4.344)
 Z-values   7.828                    4.250
 P-values   0.000                    0.000

    CUBES = 1.286*Visual, Errorvar.= 14.267, R² = 0.104
 Standerr  (0.253)                  (1.836)
 Z-values   5.083                    7.770
 P-values   0.000                    0.000

 LOZENGES = 4.484*Visual, Errorvar.= 42.107, R² = 0.323
 Standerr  (0.613)                  (5.263)
 Z-values   7.318                    8.001
 P-values   0.000                    0.000

 PAR COMP = 2.819*Verbal, Errorvar.= 2.454 , R² = 0.764
 Standerr  (0.213)                  (0.497)
 Z-values   13.219                   4.940
 P-values   0.000                    0.000

 SEN COMP = 4.021*Verbal, Errorvar.= 5.978 , R² = 0.730
 Standerr  (0.285)                  (0.891)
 Z-values   14.111                   6.713
 P-values   0.000                    0.000

 WORDMEAN = 6.725*Verbal, Errorvar.= 15.573, R² = 0.744
 Standerr  (0.492)                  (3.166)
 Z-values   13.661                   4.919
 P-values   0.000                    0.000

 ADDITION = 16.335*Speed, Errorvar.= 299.625, R² = 0.471
 Standerr  (1.501)                  (37.516)
 Z-values   10.880                   7.987
 P-values   0.000                    0.000

 COUNTDOT = 15.881*Speed, Errorvar.= 75.403 , R² = 0.770
 Standerr  (1.303)                  (26.471)
 Z-values   12.186                   2.849
 P-values   0.000                    0.004

 S-C CAPS = 29.703*Speed, Errorvar.= 467.122, R² = 0.654
 Standerr  (2.050)                  (86.560)
 Z-values   14.491                   5.397
 P-values   0.000                    0.000
```

The RMSEA point estimate is larger and the chi-square value for this method is 57.921, compared to 49.72 for the previous method.

```
Goodness-of-Fit Statistics

 Degrees of Freedom for C(1),C(6)                    24
 Weighted Least Squares Chi-Square (C1)              57.921 (P = 0.00012)
 Yuan-Bentler (1997) Chi-Square for C1 (C6)          41.306 (P = 0.01541)

 Estimated Non-centrality Parameter (NCP)            33.921
 90 Percent Confidence Interval for NCP              (15.371 ; 60.165)
```

```
 Minimum Fit Function Value                              0.402
 Population Discrepancy Function Value (F0)              0.236
 90 Percent Confidence Interval for F0                   (0.107 ; 0.418)
 Root Mean Square Error of Approximation (RMSEA)         0.0991
 90 Percent Confidence Interval for RMSEA                (0.0667 ; 0.132)
 P-Value for Test of Close Fit (RMSEA < 0.05)            0.00867

Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -4.443
   Median Standardized Residual =   -0.595
  Largest Standardized Residual =    3.927

 Stemleaf Plot

 - 4|44
 - 3|91
 - 2|99777522
 - 1|7775432210
 - 0|654310
   0|12237
   1|1124889
   2|048
   3|59
 Largest Negative Standardized Residuals
 Residual for         SEN COMP and         VIS PERC  -4.426
 Residual for         SEN COMP and         LOZENGES  -2.716
 Residual for         WORDMEAN and         VIS PERC  -2.729
 Residual for         ADDITION and         VIS PERC  -3.098
 Residual for         ADDITION and         LOZENGES  -2.936
 Residual for         ADDITION and         WORDMEAN  -2.723
 Residual for         COUNTDOT and         PAR COMP  -4.443
 Residual for         COUNTDOT and         SEN COMP  -2.856
 Residual for         COUNTDOT and         WORDMEAN  -3.872
 Largest Positive Standardized Residuals
 Residual for         VIS PERC and         VIS PERC   3.927
 Residual for         LOZENGES and            CUBES   3.490
 Residual for         S-C CAPS and         VIS PERC   2.780
```

The distribution of the standardized residuals for this method is different from those obtained with the two ML methods, as illustrated by the stem-and-leaf plot.

6. Comparing results

The values of the goodness-of-fit statistics for the three methods considered are summarized in the table below. Maximum likelihood estimation using the asymptotic covariance matrix (robust maximum likelihood estimation) yielded a smaller Chi-square value and RMSEA point estimate that those obtained with the other two methods. However, all three methods suggest that the model is not supported by the data.

Value	Method 1 (ML)	Method 2 (ML with ACM)	Method 3 (WLS)
Chi-square	51.19	49.72	57.92
Degrees of freedom	24	24	24
P-value	0.00100	0.00154	0.00012
RMSEA	0.089	0.086	0.099

Turning to the residuals estimated under the three methods, we compare the summary statistics for the standardized residuals.

Method 1:

```
Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -3.020
   Median Standardized Residual =    0.000
  Largest Standardized Residual =    4.965

 Stemleaf Plot

 - 3|00
 - 2|9311
 - 1|7322
 - 0|876552110000000000
   0|111336689
   1|248
   2|139
   3|
   4|5
   5|0


Largest Negative Standardized Residuals
 Residual for         ADDITION and         LOZENGES  -2.973
 Residual for         COUNTDOT and         PAR COMP  -3.020
 Residual for         S-C CAPS and         COUNTDOT  -2.904

Largest Positive Standardized Residuals
 Residual for         COUNTDOT and         ADDITION   4.965
 Residual for         S-C CAPS and         VIS PERC   4.495
 Residual for         S-C CAPS and         SEN COMP   2.904
```

Method 2:

```
Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -3.292
   Median Standardized Residual =    0.000
  Largest Standardized Residual =    3.957

 Stemleaf Plot

 - 3|31
 - 2|1000
 - 1|642
 - 0|97753210000000000000
   0|1123447889
   1|47
   2|35
   3|1
   4|0
 Largest Negative Standardized Residuals
 Residual for         ADDITION and         LOZENGES  -3.053
 Residual for         S-C CAPS and         COUNTDOT  -3.292

 Largest Positive Standardized Residuals
```

```
 Residual for         COUNTDOT and         ADDITION   3.054
 Residual for         S-C CAPS and         VIS PERC   3.957
```

Method 3:

```
Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -4.443
   Median Standardized Residual =   -0.595
  Largest Standardized Residual =    3.927

 Stemleaf Plot

 - 4|44
 - 3|91
 - 2|99777522
 - 1|7775432210
 - 0|654310
   0|12237
   1|1124889
   2|048
   3|59
 Largest Negative Standardized Residuals
 Residual for         SEN COMP and         VIS PERC  -4.426
 Residual for         SEN COMP and         LOZENGES  -2.716
 Residual for         WORDMEAN and         VIS PERC  -2.729
 Residual for         ADDITION and         VIS PERC  -3.098
 Residual for         ADDITION and         LOZENGES  -2.936
 Residual for         ADDITION and         WORDMEAN  -2.723
 Residual for         COUNTDOT and         PAR COMP  -4.443
 Residual for         COUNTDOT and         SEN COMP  -2.856
 Residual for         COUNTDOT and         WORDMEAN  -3.872

 Largest Positive Standardized Residuals
 Residual for         VIS PERC and         VIS PERC   3.927
 Residual for         LOZENGES and            CUBES   3.490
 Residual for         S-C CAPS and         VIS PERC   2.780
```

The distribution of the standardized residuals for this method is different from those obtained with the two ML methods, as illustrated by the respective stem-and-leaf plots. Those of the first two models show an approximate normal distribution for the residuals, but this is not the case for the model estimated by WLS. The standardized residuals for method 1 have a range of 7.985, for model 2 it is 7.249, and for model 3 it is 8.370. The standardized residuals for all three methods are large which indicates that the model is not supported by the data.