# Standardized solutions and direct, indirect, and total effects

## 1. Standardized solutions

In LISREL there are two kinds of standardized solutions: SS (Standardized Solution) and SC (Completely Standardized Solution). In the SS solution, the latent variables are scaled to have standard derivations equal to unity; the *observed* variables are left in their original metric. In the SC solution, the observed as well as the latent variables are standardized.

These standardized solutions can only be obtained after the model has been fitted with the variables in their original metric. After that, the standard deviations of the observed and latent variables are estimated and applied as scale factors in the rows and columns of the estimated parameter matrices.

## 2. Direct, indirect and total effects

The path diagram in Figure 1 includes both direct and indirect effects of $\xi_1$ on $\eta_2$. For example, in addition to the direct effect $\gamma_{21}$ of $\xi_1$ on $\eta_2$ there is an indirect effect $\beta_{21}\gamma_{11}$ mediated by $\eta_1$. Similarly, although there is no direct effect of $\xi_3$ on $\eta_1$, there is an indirect effect $\beta_{12}\gamma_{23}$ mediated by $\eta_2$.

There are never direct effects of an $\eta$ on itself, *i.e.*, all diagonal elements of $\mathbf{B}$ are zero. Nevertheless, there may be a total effect of each $\eta$ on *itself*. But this can only occur in non-recursive models and can best be understood by defining a cycle. A cycle is a causal chain going from one $\eta$, passing over some other $\eta$'s, and returning to the original $\eta$. The two $\eta$'s in Figure 1 are shown in isolation in Figure 2.

**Figure 1: Path diagram for Hypothetical Model**

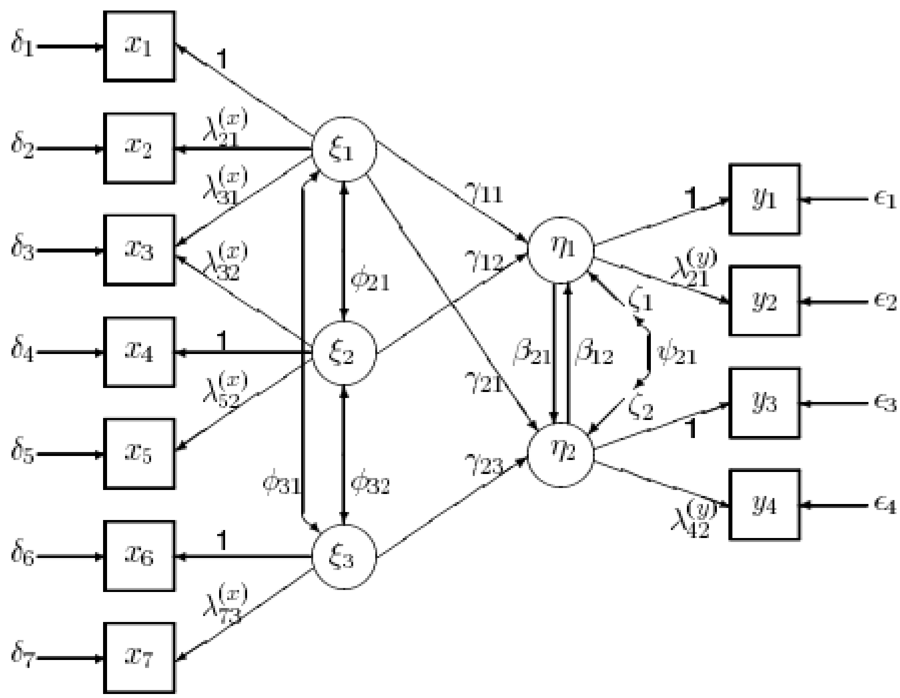
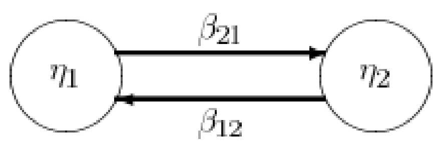


**Figure 2: Reciprocal causation between $\eta_1$ and $\eta_2$**



One cycle for $\eta_1$ consists of one path to $\eta_2$ and a return to $\eta_1$. The effect of one cycle on $\eta_1$ is $\beta_{21}\beta_{12}$. After two cycles the effect will be $\beta_{21}^2\beta_{12}^2$, after three cycles $\beta_{21}^3\beta_{12}^3$, etc. The total effect on $\eta_1$ will be the sum of the infinite geometric series $\beta_{21}\beta_{12} + \beta_{21}^2\beta_{12}^2 + \beta_{21}^3\beta_{12}^3 + \ldots$, which is $\beta_{21}\beta_{12} / (1 - \beta_{21}\beta_{12})$ for $\beta_{21}\beta_{12} < 1$.

In general, the total effect of $\boldsymbol{\eta}$ on itself is

$$\mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \cdots = (\mathbf{I} - \mathbf{B})^{-1} - \mathbf{I}, \tag{1}$$

provided the infinite series converges. Similarly, one finds that the total effect of $\boldsymbol{\xi}$ on $\boldsymbol{\eta}$ is

$$(\mathbf{I} + \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \cdots)\boldsymbol{\Gamma} = (\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\Gamma}. \tag{2}$$

A necessary and sufficient condition for convergence of the series in (1) and (2), *i.e.*, for stability of the system, is that all the eigenvalues of **B** are within the unit circle. In general, the eigenvalues of **B** are complex numbers somewhat difficult to compute. However, a *sufficient* condition for convergence is that the largest eigenvalue of $\mathbf{BB}'$ is less than one, and this is very easy to verify. The program prints the largest eigenvalue of $\mathbf{BB}'$ under the name STABILITY INDEX.

General formulas for indirect and total effects are given in Table 1.

**Table 1: Decomposition of effects**

| | $\boldsymbol{\xi} \rightarrow \boldsymbol{\eta}$ | $\boldsymbol{\eta} \rightarrow \boldsymbol{\eta}$ |
|---|---|---|
| Direct | $\boldsymbol{\Gamma}$ | $\mathbf{B}$ |
| Indirect | $(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\Gamma}-\boldsymbol{\Gamma}$ | $(\mathbf{I}-\mathbf{B})^{-1}-\mathbf{I}-\mathbf{B}$ |
| Total | $(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\Gamma}$ | $(\mathbf{I}-\mathbf{B})^{-1}-\mathbf{I}$ |
| | $\boldsymbol{\xi} \rightarrow \mathbf{y}$ | $\boldsymbol{\eta} \rightarrow \mathbf{y}$ |
| Direct | $\mathbf{0}$ | $\boldsymbol{\Lambda}_y$ |
| Indirect | $\boldsymbol{\Lambda}_y(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\Gamma}$ | $\boldsymbol{\Lambda}_y(\mathbf{I}-\mathbf{B})^{-1}-\boldsymbol{\Lambda}_y$ |
| Total | $\boldsymbol{\Lambda}_y(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\Gamma}$ | $\boldsymbol{\Lambda}_y(\mathbf{I}-\mathbf{B})^{-1}$ |